Computationally enhanced, quantum-optical ways of achieving axial super-resolution in Optical Coherence Tomography

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ABSTRACT

Mechanisms enabling axial super-resolution in OCT are identified and described based on the available research in this area. The limitations of using each mechanisms are discussed.

1. INTRODUCTION

Being able to probe the structure of an object with more detail enables more comprehensive studies of all kinds of natural phenomena and processes. This is why, in any kind of imaging, be it optical, ultrasound or ionising, achieving better resolution is a goal in itself, one which has been pursued despite the presence of the fundamental barriers. The persistent, tireless push to go beyond the fundamental limitations culminated in the creation of super-resolution imaging, with its authors awarded the Nobel Prize in recognition of the merit of their successful pursuit [1]. The proposed super-resolution imaging techniques mostly concern microscopy and improving the lateral resolution. Although inventive and truly imaginative, they cannot be directly translated to depth imaging performed by Optical Coherence Tomography (OCT) due to it being governed by different rules. Whereas the lateral resolution is related to the frequency of light and the numerical aperture of the imaging lens, the depth resolution in OCT depends nearly solely on the frequency range. This is why, OCT requires different super-resolution approaches exploiting the unique nature of OCT depth image formation.

Here, a representative selection of works is discussed where attempts are reported at achieving axial super-resolution in OCT. The presented approaches – Spectral Estimation [2], multi-rate [3], quantum [4] and our own quantum-mimic OCT [5] – are considered in terms of the mechanisms underlying the depth image formation that they use to achieve the improvement. The specific mechanisms are identified in each research, described more broadly in relation to OCT imaging and characterized in terms of their limitations.

2. AXIAL SUPER-RESOLUTION TECHNIQUES

The signal in Optical Coherence Tomography (OCT) can be described as

$$S_{\mathrm{OCT}}(k) = 2I_0(k)|1+f(k)|^2, \text{ (1)}$$

where I_0 is the light source's spectrum, and f is an object's transfer function which describes the phase delays which the object imparts on the light, and k – the light's wavenumber.

For a two-interface object, i.e. the simplest object used to consider the axial resolution limitations, the transfer function is

$$f(k) = r_1\exp(i2kL) + r_2\exp\big(i2k(L+\Delta L)\big), \text{ (2)}$$

where r_1, r_2 are reflectivities of the object interfaces, L is the distance from the interferometer zero delay point to the first interface, and ΔL is the distance between the interfaces. For clarity, we omitted refractive indices and chromatic dispersion of any of the layers. Substituting (2) in (1) gives the basic expression for an OCT signal (the term corresponding to an autocorrelation peak, $r_1r_2\cos(2k\Delta L)$, is omitted):

$$S_{\mathrm{OCT}}(k) \approx 2I_0(k)(1 + r_1\cos(2kL+\varphi_1) + r_2\cos(2k(L+\Delta L)+\varphi_2) + \cdots) = 2I_0(k)\big(1+M(k)\big). \text{ (3)}$$

where M is a modulatory part representing the fringes observed in the spectrum, and φ_1, φ_2 are constant phase components determining the cosine's phase shift in the k space. In very simple terms, an OCT signal can be viewed as a series of cosines overlapping the light source's spectrum. Each cosine represents a different interface and has a frequency equal to the position of that interface.

Spectral Estimation OCT [2]. In this kind of algorithms, the frequencies present in the spectrum are estimated with a higher precision than what Fourier transformation allows in the traditional approach. These algorithms work around the signal-shape-related limitations of Fourier transformation that result in at least two OCT-performance-suppressing co-dependencies: bandwidth-resolution and phase-resolution.

(bandwidth-resolution limit) The peak width in the Fourier transform (so the axial resolution of the A-scan) is smaller for broader input signals (so a broader spectrum). This is why, rectangle-shaped spectra provide better axial resolution than Gaussian-shaped spectra, which fits into the premise that the more of the signal is had, the surer one is about its contents. The Spectral Estimation algorithms ignore the spectral shape and focus instead solely on the modulatory part M (eq. 3), i.e. the fringes, of the OCT signal.

(phase-resolution limit) The precision of frequency decomposition depends on the signal phase shift. Governed by the constant phase difference, $\varphi_1 - \varphi_2$, in OCT, it leads to the translation of the fringes in the acquired spectrum. Consequently, Fourier transformation is able to resolve two close interfaces even if their distance is smaller than the Fourier transform peak width dictated by the input signal width. It is best illustrated on a two-interface object for which the signal's modulatory part, M, becomes a beating pattern (Fig. 1, centre column). A different $\varphi_1 - \varphi_2$ leads to a different location of the waist of that fringe pattern in the signal. This results in better resolving of the two frequencies comprising the fringes, and consequently, in seemingly better axial resolution (Fig. 1, right-hand side). The Spectral Estimation algorithms, due to focusing on the fringes themselves, are to a great extent immune to the constant phase difference. This is why, depending on how "lucky" one was with the constant phase difference, the true resolving power increase of the Spectral Estimation algorithms with respect to Fourier transformation varies between 2 and 10, as was practically showed by its authors **[6]**.

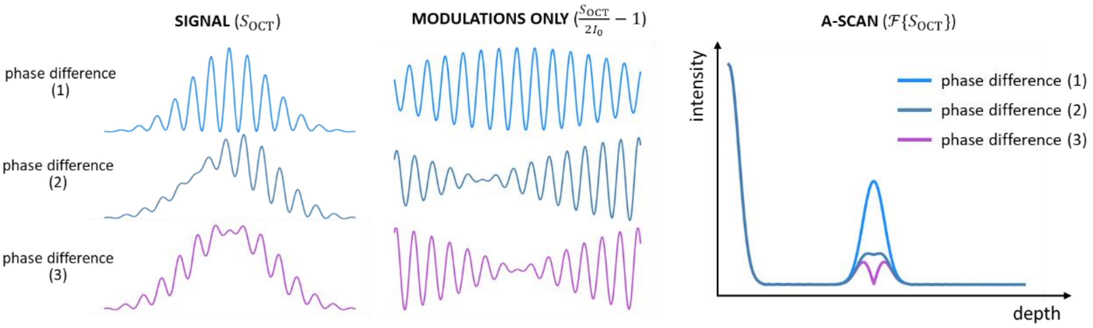


Figure 1. Depending on the initial phase, two interfaces are resolved even if their distance is smaller than the axial resolution.

Quantum OCT. In quantum OCT (q-OCT), entangled photon pairs – generated in a nonlinear crystal – propagate in a Mach-Zehnder-like interferometer: one photon in the object arm, the other photon in the reference arm. They meet at the beamsplitter whose two output ports are monitored by single-photon detectors. The coincidence rate of the two photons arriving each at a different detector is the q-OCT signal providing structural information about the imaged object. Q-OCT is very well known for its twice better axial resolution which is a direct result of the two photons being intrinsically correlated with each other.

This fact is evident when one considers the formula for the q-OCT signal:

$$S_{\mathrm{q-OCT}}(k_1, k_2) = I_0(k_1, k_2)(|f(k_1)|^2 + |f(k_2)|^2 - 2\mathrm{Re}\{f(k_1)f^*(k_2)\}), (4)$$

where wavenumbers k_1, k_2 identify photons in the pair, and the term $f(k_1)f^*(k_2)$ represents the autocorrelation.

Substituting (2) in (4) and omitting all the terms but the autocorrelation one gives:

$$S_{\mathrm{q-OCT}}(k) \approx I_0(k)\big(1 + r_1^2\cos(4kL) + r_2^2\cos\big(4k(L + \Delta L)\big) + \cdots\big), (5)$$

where $k_1 = k_0 + k$ and $k_2 = k_0 - k$ were used to ensure entanglement of the photons.

Comparing the expressions for the OCT signal (3) and q-OCT signal (5), one sees how the axial resolution is improved twice: while the OCT cosines incorporate the factor of 2, in the q-OCT cosines, this factor is twice larger, 4. In simple terms, this means that in q-OCT the same interfaces are represented by cosines whose frequency is twice as high. This leads to the peaks in the Fourier transform corresponding to these interfaces to be placed at twice the original distance, and consequently, the separation of these interfaces to be twice as big.

While 2-photon entanglement provides 2-fold resolution increase, 3-photon entanglement provides 3-fold resolution increase as was theoretically shown [7]. Generally, N-photon entanglement will theoretically lead to N-fold resolution increase. There are two practical obstacles on this possible course of action. One, the q-OCT signal incorporates parasitic frequencies which show as artefacts in the Fourier transform, effectively scrambling the high-resolution image. The more photons that are entangled, so the bigger the resolution increase, the more artefacts. Two, the biggest hurdle, even for photon pairs, the amount of light is not enough to image non-invasively anything more complicated than pieces of glass.

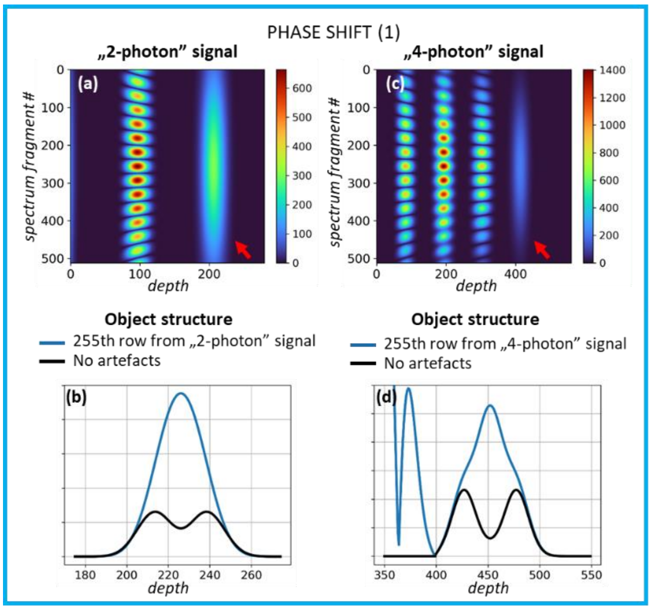
Quantum-mimic OCT. The problem with insufficient amount of light is solved in quantum-mimic OCT (qm-OCT) [5]. There, a traditional OCT signal is mathematically autocorrelated, imitating the correlation of photons in the entangled photon pair. Although with even more artefacts than its quantum counterpart, the quantum-mimic signal shows resolution improvement, but this improvement is smaller due to the spectrum shape.

The artefacts, whether in q-OCT or in qm-OCT, are what obstructs achieving the resolution increase otherwise dictated by how many photons are autocorrelated during the entanglement process, as seen in Fig. 2. It can be shown that in practice, the presence of artefacts reduces the resolution increase to the levels obtained in Spectral Estimation OCT.

Multi-rate OCT. In the multi-rate approach, the carrier frequency of the fringes is slightly shifted to induce beating in the signal constructed by summing the original signal with its shifted version. Realised experimentally by using a phase modulator [3] or algorithmically by the multiplication by a phase factor [8], it allowed 2 to 7 times axial resolution increase.

This approach artificially induces beating associated with the presence of two close frequencies and therefore, if the parameters are not selected properly, it can falsely resolve two interfaces in places where there is only one.

Inducing the beating in the signal can be viewed as artificial, forceful increase of frequency of the modulations and in this respect, multi-rate OCT bears some resemblance to q-OCT and qm-OCT which both use this resolution-improving mechanism directly.



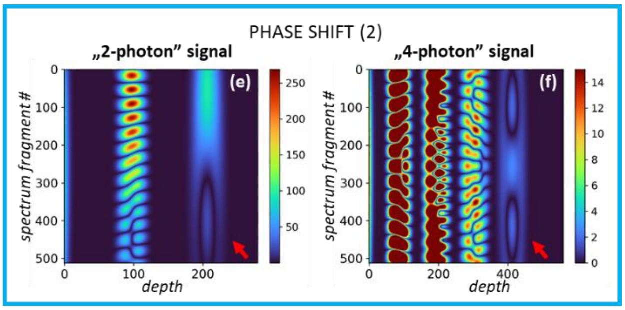


Figure 2. 2-photon and 4-photon signals obtained by mimicking 2- and 4-photon entanglement show artefacts (modulated vertical elements) together with structural elements (smoother vertical elements marked with red arrows). For the spectrum representing PHASE SHIFT (1) in Fig. 1, artefacts in both **(a)**, **(b)** 2-photon and **(c)**, **(d)** 4-photon case suppress the high-resolution information. The spectrum representing PHASE SHIFT (2) in Fig. 2 gives **(e)** 2-photon and **(f)** 4-photon signals where the superposition of the structure and the artefact is clearly seen.

3. SUMMARY AND DISCUSSION

An OCT A-scan is obtained by determining the frequencies of modulations present in the detected spectrum since these frequencies are in the direct relationship with the position of the reflectors in the object. Consequently, improving the axial resolution boils down to improving the precision of the frequency decomposition of the OCT spectrum. The presented selection of works on axial super-resolution shows two distinct mechanisms enabling heightened frequency decomposition: decoupling its precision from the signal shape (Spectral Estimation OCT) and increasing the frequency of the modulations in the signal, either directly through autocorrelation (q-OCT and qm-OCT), or indirectly through carrier frequency shift (multi-rate OCT). Whereas the former removes the resolution-suppressing constraints, the latter aims to boost the certainty of the information content. Interestingly, both mechanisms show the same kind of maximum resolution increase. While the first one is inherently limited by what is present in the signal once the resolution-suppressing factors are removed, the second one – inherently able to provide infinite resolution increase – is suppressed by its own by-products.

One could think of a third mechanism which has not been explored yet, most probably to the mere impossibility of its practical implementation, but which theoretically is able to provide an infinite increase as well. Inspired by multi-rate OCT, where the carrier frequency is shifted, a technique could be designed where the envelope is shifted instead. In this way, the fringe pattern at optical frequencies exceeding the spectral range of the light source could be pushed towards that spectral range and consequently, visualized. Stitching of such frames together would form an ultra-high resolution signal.

4. REFERENCES

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