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Book Reviews

The Book Reviews section basically intends to promote books written in English. However, as there are lots of interesting books written in native languages nowadays, that are worth to be presented to a broader community of logicians and philosophers, we are widening the scope of the section starting with a review of a Polish publication. At the same time we would like to encourage our readers to submit reviews of books written in languages other than English.

The Editors

Tomasz Jarmużek and Marcin Tkaczyk, **Normalne logiki pozycyjne** (Normal Positional Logics), Wydawnictwo KUL, Lublin (Poland), 2015, 202 pages, ISBN 978-838061-114-6.

The monograph *Normalne logiki pozycyjne* (in English: Normal Positional Logics; in short: NPL) is a contribution and an introduction to the field of positional logic — the logic that might be recognized as the third way between hybrid and modal logic. It is probably the first publication which widely treats of results of scientific research of Polish logician Jerzy Łoś and of possible development of logical systems which were subject of his considerations. As authors emphasize the last word about such logic has not been said yet. Surely the work of Jarmużek and Tkaczyk is also a forerunner of further developments of investigations begun by Łoś.

What is a positional logic? The authors in introduction describe it as a logical tool, which might be located between hybrid and modal logic. It is so since points, to which we relativize logical value might be recognized as terms of a language of positional logic. Differently than in modal logic, where points appeared only in a metalanguage. But also

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differently than in hybrid logic, where so called nominals, which denote points, are atomic expressions and with proper connections may with others expressions formulate complex expressions of the language:

> In positional logic we try to strengthen the language by enabling use of it for speaking about points of relativization, also known as positions. Unlike in hybrid logic we do not agree to treat the symbols introduced for such purpose as sentential expressions. For us it is enough to treat them as usual terms. NPL, p. 8 (transl. by M. Klonowski)

Moreover they introduce to the language a connective, which asserts that expressions of language are realizable (for instance: true) in a given point. The authors call it "connective of realization" and denote by the letter "$\mathcal{R}$". The operator $\mathcal{R}$ is a sentence-forming functor of both sentential and nominal arguments, and with a term α and an expression φ, which authors call "quasi-expression", it formulates an expression $\mathcal{R}_\alpha\varphi$. Such expression asserts that the expression φ is realizable at the point α. As we can see it is a modal operator, which means that it co-determines the way in which sentences hold. By means of such operator we can easily define others, more complex modal operators.

How we should interpret operator $\mathcal{R}$? It is rather difficult to disagree with authors who claim that possibilities of interpretations are practically endless, which makes positional logic an interesting and useful tool.

After such explanations the authors clarify what they understand by normality in context of positional logic. For this purpose they refer to a system of positional logic **MR** and its extensions. Basically, in a normal positional logic all Boolean connectives have in all contexts (points denoted by terms) a standard meaning. This means that in all positions classical logic holds. For example conjunction is true at a given point if and only if its conjuncts are true at this point. Name *normal positional logic* can bring to mind normal modal logic. One of the most important result of the monograph is a successful attempt of showing an analogy between normal positional and normal modal logic.

The book contains seven chapters. In the first one we will find an elementary information from set theory, classical propositional logic (in short: **PL**) and first-order predicate logic (in short: **FOL**) as well as arrangements concerning notation.

The second chapter has a historical character and presents stages of development of positional logic. The main goal of it is to introduce work of Łoś as well as to show that positional logic takes its beginning in his work from late 40s.

Łoś in his master thesis, written in 1947 (cf. [6]), presents probably the first system of temporal logic. In this work he uses the operator of realization denoting it by **U**. Whereas denotation by the big letter "R" was first introduced by Rescher. The authors state that considerations about positional logic might be found, for example, in important works of Rescher and Urquhart form 60s and 70s [11, 12]. Łoś paper was cited in these works and his logic was presented in a review by Henryk Hiż [1] on the pages of *Journal of Symbolic Logic*. Unfortunately, nowadays it is not recognized as the first paper about temporal aspects of reasoning which was the main motivation of Łoś.

The authors might be treated as continuers of Łoś's approach as series of their articles show: [2, 3, 4, 13, 14]. But there are also others logicians who find subject of propositional logic interesting (for example, [7, 8, 9, 10]).

In the second chapter authors also discuss a temporal interpretation of positional logic and in particular the system described by Łoś. A goal of Polish logician was simple, he tried to elaborate an axiomatization of part of a physical language (he focused, inter alia, on a problem of including time coordinates in reasoning). The system of logic that he developed was based on **PL** with quantifiers which bound variables of all category, in particular sentential variables.

In the third chapter a reader will find details of the system **MR**. Results from this chapter where partially presented in [4]. But there are also new results that have been shown. For instance the authors present not only a notion of usual model but also of a general model. The difference might be briefly explain in following way: in the general model we can restrict determination of logical values formulae build by Boolean functors to points which are referents of individual constants. From that point of view usual models are redundant (pp. 50–60). However the authors show that semantics consequence relations determined modulo usual models and general models are extensionally equal (Theorem 3.2.1, p. 54). These two notions of models can be attributed to different use. For analyses of extensions of **MR** usual models are more useful. For instance we can get extension by adding quantifiers by which we want to refer to positions which are not named by individual constants. On the other hand on the ground of non-normal positional logic general models are much more useful. Also they are more useful for the transition to a simplified semantic of **MR**. In the third chapter authors present an axiomatization of **MR** as well (pp. 60–63). They prove by logical matrices that such axiomatization is independent, where independence is understood as in the case of normal modal logic (pp. 70–71). By virtue of the proof presented in [4] they recognize soundness and completeness of **MR** (Theorem 3.3.1, p. 71).

In the fourth chapter a reader will find an alternative semantic approach to system **MR**. The authors show simplified semantics, which is based on functions that assign logical values. What is important for such semantic we choose some subset of all valuation functions. These functions should allow us to determine semantic consequence relation of **MR**. We describe such subset by specific conditions (pp. 73–75). They also analyse a relationship between these conditions (pp. 75–78). For our purpose we call such functions mr-valuations. In the chapter the reader also can find proof of independence of axioms in regard to mr-valuation (Theorem 4.2.1, p. 78). Moreover a proof of soundness different than in [4] has been presented (pp. 91–92). In the proof the authors use some other results which asserts one to one relationship between mr-valuations and general models. There are also some new conditions introduced which with some of discussed conditions determine the set of mr-valuations (Theorem 4.2.4, pp. 94–100). The authors consider mr-valuations described by latter conditions since they help them to prove completeness theorem by embedding **MR** into **PL** (Theorem 4.2.8, p. 112), also to introduce a similar way of checking validity of formulae as in the case of **PL** (pp. 104–109), to formulate an alternative axiomatization and to describe a tableau approach to **MR** as well.

In the fifth chapter we can find out about alternative axiomatizations of **MR**. One of such axiomatizations leads to description of normality of positional logic. Which means that it helps to form conditions which normal positional logic must fulfil (pp. 117–118). In this chapter also some ideas of extensions of **MR** has appeared. First one is based on an iteration of operator $\mathcal{R}$. The authors show and discuss two approaches to the iteration, barren (pp. 120–121) and relevant (pp. 121–123). In the case of the barren iteration logical value of a formula does not change when we add another operator $\mathcal{R}$ to the formula, in the case of the relevant iteration situation is different. But there are other ways of extending **MR** discussed by the authors. For instance we can extend the set of expression of positional logic to a set of formulae in which there is no operator of realization. More specifically any formula of **PL** might become a formula of **MR**. Values of such formulae might be needed at chosen position (pp. 125–126) or at all positions in model (pp. 126–128). In the chapter also the tableau approach to **MR** has been presented (pp. 129–131).

In the sixth chapter system **MR** is strengthened by quantifiers to system **MRQ**, which contains **FOL**. In **MRQ** quantifiers may bound positions, we can also determine relationship between positions as well as operations on positions. **MRQ** might be considered as a sum, in some sense, of **MR** and **FOL**. In such system we can refer to positions not only by individual constants but also by complex terms and function symbols. Models of **MRQ** are usual models described in the third chapter. Axioms of **MRQ** are axioms of **FOL** and three specific axioms of **MR**. All theses of **MR** appeared to be theses of **MRQ** (Lemma 6.2.3, p. 138). Also the soundness theorem has been shown (Theorem 6.3.1, p. 139). Probably the most interesting result is completeness theorem proved by embedding in **FOL** (Theorem 6.3.2, p. 145). (Description of useful functions and lemmas for the proof of completeness a reader can find on pages 140–150.) It should be pointed out, as the authors did, that **MRQ** is conservative extension of **MR**, which was also discussed in [4] in Theorem 6.1:

> Theorem 6.1 says that there are not any specific, new theorems expressed in the old language, that cannot be proved in the system **MR**. [4, p. 160]

Of course it does not mean that there is no way of extending **MRQ**. For instance, we can consider occurrences of formulae of **MRQ** in a range of the operator $\mathcal{R}$. Another idea, we can let pure formulae of **PL** without $\mathcal{R}$, but since we already have predicates in language this seems to be redundant. The authors specify this idea assuming that some predicates might express properties of positions, while formulae of **PL** express something about these things which are relativized to positions. Which lead to idea of two domains: domain of positions and domain of objects.

In the seventh chapter we will find out about some applications of positional logic. The authors present some reconstructions of the famous Master Argument and also how we can express normal modal systems such as **K**, **T**, **B**, **S4** and **S5** in **MRQ**. The reconstruction of modal systems demands to give a semantic characteristic of them by expressing constraints of accessibility relation in the language of **FOL**. Also some discussion of rules of the standard translation from expressions of modal logic to expressions of **FOL** and also of the method of Sahlqvist algorithms have been presented (pp. 184–185).

After short repetition of basic information about modal logic the authors present appropriate constructions. Firstly they remind that in the language of **MRQ** we have suitable equivalents of the most important conceptual tools of the semantics of modal logic. For example by the operator $\mathcal{R}$ we can express relativization of logical values to some positions, and an equivalent of accessibility relation might be formulated by a binary predicate. In that way we are able to introduce definitions of modal connectives in range of the operator $\mathcal{R}$. These definitions are counterparts of truth conditions for formulae build with modal connectives. Proofs of soundness (pp. 178–183) and completeness (pp. 187–189) of systems of modal logic on the ground of **MRQ** are also presented. The authors are aware that they could use Sahlqvist algorithms in proof of completeness. But the goal was to use more elementary tools, which are completely contained within system **MRQ**.

The book under review is an interesting and successful attempt of presentation of an old but forgotten paradigm of logic which can be widely applied to metalogical problems. The book should be accessible also for English readers.

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