

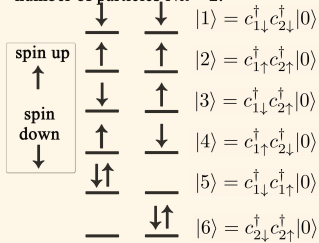
INTRODUCTION

We construct different finite size fragments of moiré triangular lattice and analyze its magnetic properties when doped away from the half-filling. In such a case one can expect a transition from a nonmagnetic state to Nagaoka ferromagnetism. This type of magnetism comes from correlations effects, or in other words, is due to constructive interference between different many-body configurations.



The illustration above displays how stacking and twisting monolayers result in localized potentials. Particles such as electrons tend to stay in this potential.

2. Configuration of lattice $N_s = 2$ with number of particles $N_{el} = 2$.



3. Construct the Hamiltonian equation with tight-binding and extended Hubbard model. This example only calculates the on-site Coulomb interaction $U = \langle ii|v|ii \rangle$.

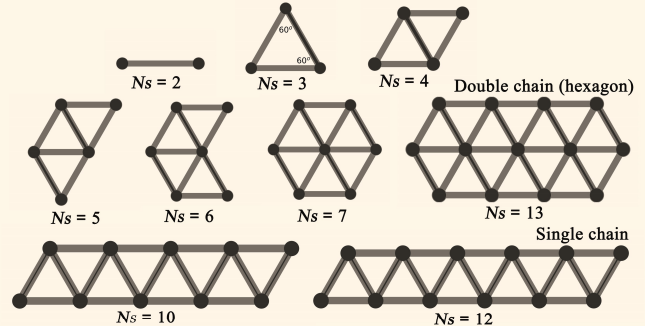
$$\hat{H} = \sum_{ij\sigma} t_{ij} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + \sum_{ijkl} \langle ij|v|kl \rangle \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma}^\dagger \hat{c}_{k\sigma} \hat{c}_{l\sigma}$$

where, the Coulomb interaction defined by

$$\langle ij|v|kl \rangle = \iint d\vec{r} d\vec{r}' \frac{\phi_i^*(\vec{r}) \phi_j^*(\vec{r}') \phi_k(\vec{r}) \phi_l(\vec{r}')}{|\vec{r} - \vec{r}'|}$$

METHODOLOGY

1. Construct the finite moiré triangular lattices, starting from the number of sites $N_s=2$ up to $N_s=13$, with a single and double chain variation.



4. Using the Exact Diagonalization method, we construct the Hamiltonian matrix for $N_s = 2$ with $N_{el} = 2$. t is the nearest neighbor energy.

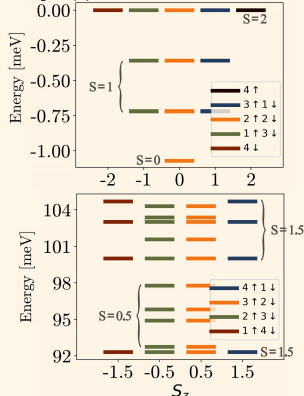
$$\hat{H} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & t_{21} & t_{12} \\ 0 & 0 & 0 & 0 & -t_{21} & -t_{12} \\ 0 & 0 & t_{12} & -t_{12} & U & 0 \\ 0 & 0 & t_{21} & -t_{21} & 0 & U \end{pmatrix}$$

We need to calculate for all possible numbers of particles on site to get the eigen values and eigen vectors.

RESULTS

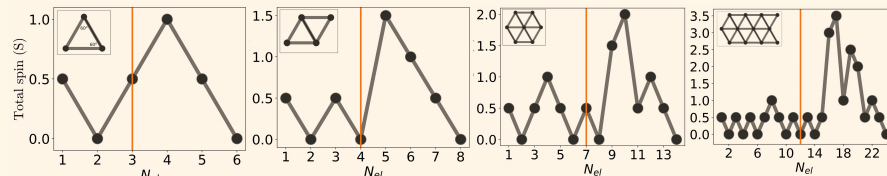
1. Spectral Energy

The spectral energy of lattice $N_s = 4$ with its quantum spin Number (S_z) for a number of particle $N_{el} = 4$ (above) and 5 (below). From here, we can specify the total spin (S).



2. Phase Diagram

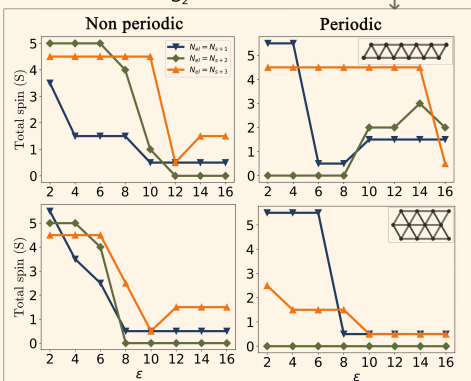
Total spin of groundstate for every possible number of particle N_{el} in the lattice site $N_s = 3, 4, 7, 12$, with $\epsilon = 10$. ϵ is a parameter related to t and U . We can see the highest total spin appears always above half-filling. This is the sign of Nagaoka ferromagnetism.



3. Spin transition

The graphs below show a spin transition happens when the total spin changes with ϵ .

By constructing $N_s = 12$ as single-chain and hexagon, we can see spin transition varying with ϵ .



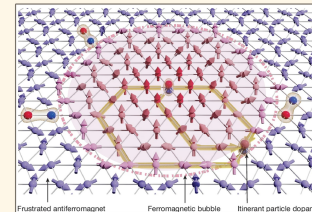
CONCLUSIONS

In summary, we observed Nagaoka ferromagnetism and Nagaoka polaron in our finite-size moiré triangular lattice when it was above half-filling.

We were able to adjust the size of the ferromagnetic bubble on the site by selecting an appropriate number of particles and a parameter ϵ .

4. Spin density

Experimental result on observing the Nagaoka polarons in a Fermi-Hubbard quantum simulator [3].



The graphs on the left display the spin density of a single-chain lattice $N_s = 12$, with $N_{el} = 13$ and $N_{el} = 14$. When the total spin is high, the particles tend to polarize on all sites. We can observe parts of the edge of the ferromagnetic bubble at a lower total spin.

REFERENCES

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- [2] Yosuke Nagaoka, *Phy.Rev.* 147, 392 (1966).
- [3] M.Lebat, M. Xu, L. H. Kendrick, A.Kale, Y. Gang, P. Seetharaman, I. Morera, E. Khatami, E. Demler, & M. Greiner, *Nature.* 629, (8011), 317-322 (2024).

ACKNOWLEDGMENT

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