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Comparison of Certain Dynamic Estimation Methods of Value at Risk on Polish Gas Market

A b s t r a c t. The paper compares the results of the estimation of VaR made using Markov chains as well as linear and non-linear autoregressive models. A comparative analysis was conducted for linear returns of the daily value of the gas base index quoted on the Day-Ahead Market (DAM) of the Polish Power Exchange (PPE) in the period commencing on January 2, 2014 and ending on April 13, 2017. The consistency and independence of the exceedances of estimated VaR were verified applying the Kupiec and Christoffersen tests.

K e y w o r d s: VaR; Markov chain; SARIMA models; GARCH models; back testing.

J E L Classification: C12, C58, G32.

Introduction

Accurate risk assessment in markets with dynamic volatility requires that real time positioning be monitored according to the frequency of observations. It is difficult in such a situation to base decisions taken in a short time horizon on the assumption that during the period under review the volatility of quotations is a sequence of independent random variables with the same distribution.

In this paper, to estimate the volatility of the gas base index quoted on the Day-Ahead Market (DAM) of the Polish Power Exchange (PPE) in the

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period from January 2, 2014 to April 13, 2017. Value-at-Risk was estimated using the following two dynamic approaches: Markov chains and autoregressive models. The aim of the study is to evaluate and compare the efficiency of VaR estimation methods using the Kupiec and Christoffersen tests for compliance and independence of exceedances.

1. Characteristics of Gas Prices

In 2012 on the Commodity Futures Market of the Polish Power Exchange (PPE), commodity futures instruments for gas appeared, and on December 31, 2012 a gas spot market was launched, where since March 2013 continuous quotations of contracts for gas supply have been announced. Figure 1.1 presents the time series of the gas_base index quoted from January 2013 (the beginning of the RDN gas operation) until April 2017. The gas_base index value corresponds to the average daily gas price [PLN/MWh] from among all transactions concluded on a given day. The index is announced every day of the week including holidays. At the beginning of the introduction of gas contracts, apart from some exceptions, gas prices remained stable. It is only at the end of 2013 that changes in the level of gas prices may be observed, as well as the trend and the seven-day cyclicality.

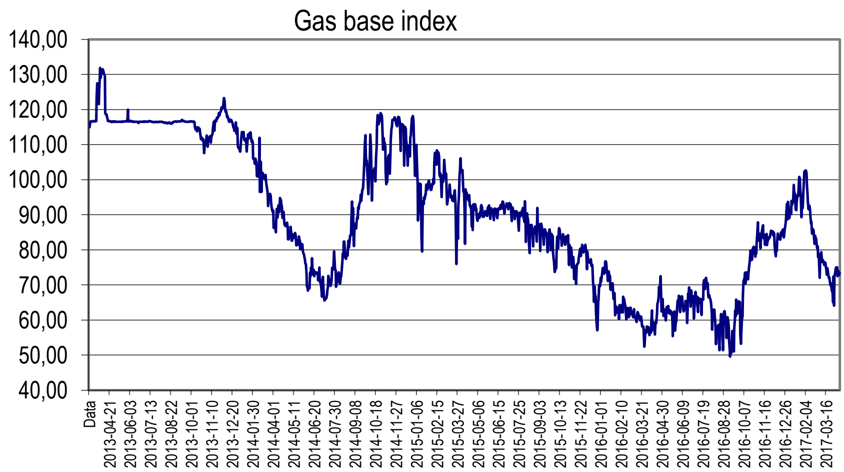

Figure 1.1. The gas_base index [PLN/MWh] quoted on the Day-Ahead Market of the Polish Power Exchange between 12 January 2013 and 13 April 2017

For further analysis a time series of daily return rates of the gas_base index was taken for the period from 01 of April 2014 to 13 of April 2017.

Figure 1.2 presents a series of return rates for the gas base index. This series clearly shows periods of very low price volatility, *i.e.*, periods of low risk of gas price changes, as well as periods of increased price volatility.

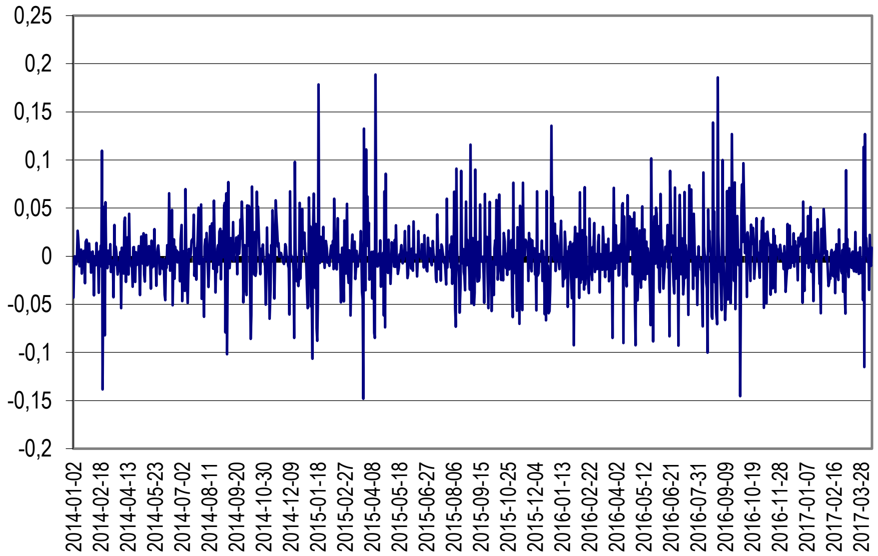

Figure 1.2. Time series of return rates of the gas_base index in the period from 02 of January 2014 to 13 of April, 2017

The basic statistical analysis allows at the level of significance of 0.05 to reject the hypothesis that the distribution of returns of gas prices is a normal distribution. The distribution assessment should take into account such characteristics as asymmetry, thick tails and leptokurticity.

2. Risk Measurement – VaR

The formal definition of VaR does not take into account the process nature of phenomena and focuses only on random variables: Value-at-Risk (VaR) represents such a loss of value that with the probability $1-\alpha$ will not be exceeded during a specified time period (Jajuga, 2000):

$$P(Y_{t+\Delta t} \leq Y_t - VaR_\alpha) = \alpha \tag{2.1}$$

where:

$\alpha \in (0,1)$ – set probability,

Δt – specified duration time of the investment,

Y_t – the present value at the moment *t*,

$Y_{t+\Delta t}$ – random variable, the value at the end of the investment.

The classical VaR valuation methods include the methods of variance – covariance, historical simulation, Monte Carlo simulation (Jajuga, 2000b) The development of financial markets is accompanied by a rapid development of the VaR measurement theory. At present, in empirical financial studies of time series, which in most cases behave as non-stationary stochastic processes, VaR estimation uses dynamic methods based on GARCH models of conditional variance (Piontek, 2002; Doman, Doman. 2009; Fiszeder, 2009; Trzpiot, 2010; Pajor, 2010; Ganczarek-Gamrot, 2006). In this paper, we will compare the results of VaR estimation taking into account the methodology of stochastic processes and the theory of Markov chains.

If Y_t represents the value at time *t*, then VaR estimation is reduced to the estimation of the distribution quantile of returns $Z_t = \frac{Y_{t+\Delta t} - Y_t}{Y_t}$. Assuming that Z_t is a stochastic process of returns characterized by the effect of concentration of volatility, the quantile of order α can be estimated as follows (Piontek, 2002; Doman, Doman, 2009):

$$Z_{\alpha\, t} = F^{-1}(\alpha)\sigma_t + \mu_t \tag{2.2}$$

where:

$F^{-1}(\alpha)$ – quantile of order α of the standardized distribution allowed for in the estimation of conditional variance σ_t^2,

σ_t^2 – conditional variance of the process,

μ_t – expected value of the process Z_t,

3. Methods of Estimation of Value at Risk

3.1 Markov Chains

Markov chains are a well-known tool used in economics (see: Ching, Ng 2006; Decewicz, 2011; Podgórska, Śliwka, Topolewski, Wrzosek, 2002; Stawicki, 2004 and many others). The Markov process with a discrete time parameter and a discrete phase space is referred to as *Markov chain*. It is defined by a sequence of stochastic matrixes of the following form:

$$\mathbf{P}(\mathrm{t}) = \lfloor p_{ij}(t) \rfloor_{r\times r}, \tag{3.1}$$

i.e., matrixes with positive elements and satisfying additional conditions in the form:

$$\forall_t \forall_i \sum_j p_{ij}(t) = 1 . \tag{3.2}$$

By denoting with $\mathbf{D}_t$ the vector of unconditional distribution of random variable Y_t, *i.e.*,

$$\mathbf{D}_t = [d_{1t}, \quad d_{2t}, \quad \cdots, \quad d_{rt}], \text{ where } d_{it} = \Pr\{Y_t = i\} , \tag{3.3}$$

we determine the probability with which the process at time t reaches the phase state i. The components of the vector $\mathbf{D}_t$ satisfy the following conditions:

$$\forall_t \forall_i \quad d_{it} \geq 0 , \tag{3.4}$$

and

$$\forall_t \quad \sum_i d_{it} = 1. \tag{3.5}$$

The dependence between unconditional distributions of random variables Y_t and Y_{t-1} is expressed by the formula resulting from the theorem on the total probability

$$\mathbf{D}_t = \mathbf{D}_{t-1} \cdot \mathbf{P}(t) . \tag{3.6}$$

Matrices $\mathbf{P}(\mathrm{t}) = \lfloor p_{ij}(t) \rfloor_{r \times r}$ reflect the mechanism of changes in the distribution of the analysed random variable Y_t over time.

Markov chain $\{Y_t, \quad t \in N\}$ with phase space $S = \{1, 2, \ldots, r\}$ is called *a homogeneous Markov chain*, if the conditional probabilities $p_{ij}(t)$ of transition from phase i to state j within a time unit, *i.e.*, in the time period from $(t-1)$ to t, do not depend on the choice of the moment t, that is

$$\forall_t \ p_{ij}(t) = p_{ij} . \tag{3.7}$$

In case of a homogeneous Markov chain the dependence (3.6) and (3.7) take the following form:

$$\mathbf{D}_t = \mathbf{D}_{t-1} \cdot \mathbf{P} . \tag{3.8}$$

Due to the nature of the data characterising the phenomenon observed, we use microdata or macrodata – these are aggregated data.

Microdata are understood as observations of an object (or multiple objects) in successive time units as well as registers of the state of the object in a given time unit. Observation of a change of state throughout the period *t–1* to *t* allows us to apply the most reliable estimator taking the following form:

$$\hat{p}_{ij} = \frac{\sum_{t=2}^{T} n_{ij}(t)}{\sum_{t=2}^{T} n_i(t-1)}, \tag{3.9}$$

where:

$$n_{ij}(t) = \begin{cases} 1 & \text{when the object at the moment } t-1 \text{ was in the state } i \\ & \text{and at the moment } t \text{ was in the state } j \\ 0 & \text{otherwise} \end{cases}$$

$$n_i(t) = \begin{cases} 1 & \text{when the object was at the moment } t \text{ in the state } i \\ 0 & \text{otherwise} \end{cases}$$

This estimator has desirable consistency properties, asymptotic unbiasedness, and has an asymptotic normal distribution of expected value

$$E(\hat{p}_{ij}) = p_{ij} \tag{3.10}$$

and variance

$$\mathrm{var}(\hat{p}_{ij}) = \frac{p_{ij}(1-p_{ij})}{\sum_{t=2}^{T} n_i(t-1)}. \tag{3.11}$$

Observation of macrodata, that is of the structure (unconditional decomposition vectors) in subsequent periods requires another apparatus that is not used in this article.

The first proposal to apply Markov chains to determine VaR was presented in Stawicki's work (2016) while presenting another decision problem. This proposal is not fully satisfactory. The article is intended to compare the results obtained by means of the proposed method and the method is recognized in scientific literature. The idea of estimating VaR at a given moment using the Markov chain model is based on the adequate construction of states. The states for the Markov chain model are suitably selected intervals which may contain the return rate.

Four states are required for the construction of Markov chain. Two of them play a special role. The first (marked as S_1) is the state of threat, taking the form of the following interval:

$$S_1 = (-\infty, -VaR)$$

and the second – the state which contains the return at the present moment.

$Z_t \in S_3$ takes the form of the following interval $S_3 = [x, y)$.

The other two states complement the entire space of the return. The state S_2 is defined as one taking the form of the interval $S_2 = [VaR, x)$, and the last state as the interval $S_4 = [y, \infty)$

Value-at-Risk is determined in accordance with the accepted rule, according to which the interval S_1 is changed empirically and thus the interval S_2, estimating at each change the matrix of the likelihood of transition to the moment when the likelihood of transition p_{31} in the matrix P is less than the assumed risk level (this work assumes $p_{31} \le 0.05$). The construction of the Markov chain described above and the estimation of its parameters, *i.e.*, the elements of the transition matrix, is a model construction closely related to the observed return Z_t. For this observation, the state S_3 is being constructed and an appropriate interval $S_1 = (-\infty, -VaR)$ is searched. The size of the interval $S_3 = [x, y)$ is dictated by the amount of available information and thus by the possibility of estimating the parameter p_{31}. In this study, the interval $[Z_t - 0{,}005, Z_t + 0{,}005)$ was accepted for each observation where the standard deviation of the examined return amounted to $STD = 0.0339$. By taking, for example, an observation of the return $Z_t = 0$, the state S_3 takes the form of the interval $S_3 = [-0.005, 0.005)$. The transition matrix (assuming the parameter $p_{31} \le 0.05$) takes the form:

		S1	S2	S3	S4
	S1	0.1207	0.2241	0.0172	0.6379
P =	S2	0.0622	0.3710	0.1866	0.3802
	S3	**0.0325**	0.3862	0.2805	0.3008
	S4	0.0365	0.3744	0.2169	0.3721

The state S_1 is presented as the interval $S_1 = (-\infty, -0.0526)$ thus indicating the Value-at-Risk = –0.0526.

By determining Value-at-Risk in this way, we obtain a simple way of making VaR dependent on the value currently observed and taking the form of the function $VaR(Z)$. In the case of a white noise process, this function is constant at the set quantile value. For the studied process, the function $VaR(Z)$ was evaluated in a parabolic form. The question remains, however, by how much the function $VaR(Z)$ changes if we determine the interval S_3 differently, and how this function is related to the type and parameters of the model generating returns. Identification of such a function gives one a simple tool for determining VaR on a current basis. For the purposes of this article, this function is estimated as a quadratic polynomial.

$$VaR(Z) = -7.274Z^2 + 0.221Z - 0.0474$$

$$R^2 = 0.5066$$

This function is presented in Fig. 3.1.

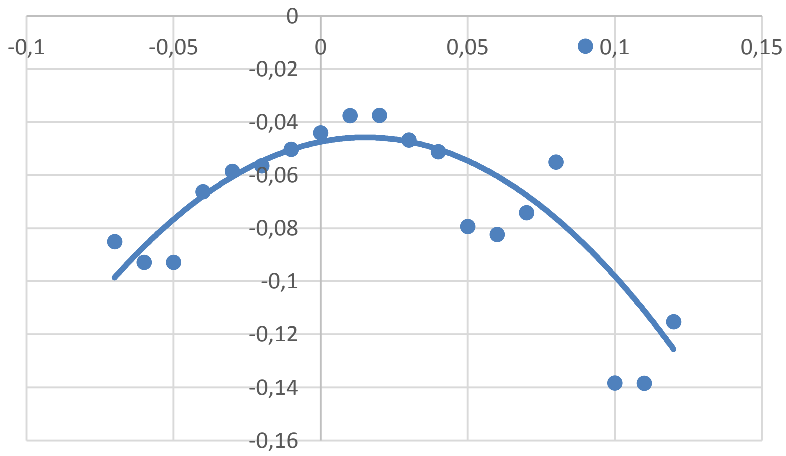

Figure 3.1. The VaR function based on the return

Figure 3.2. presents a selected part of a time series of returns (z_t) and the estimated $VaR_{0.05}$ for the one-day investment horizon using the theory of Markov chains.

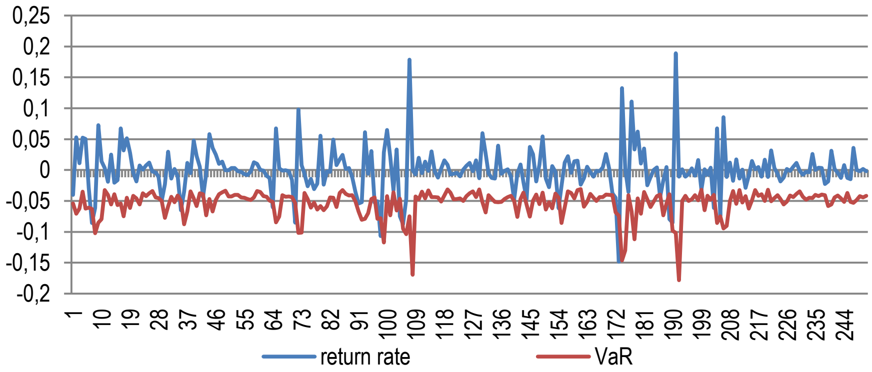

Figure 3.2 The results of VaR estimation for the selected subperiod of 250 observations using Markov chains

3.2 Autoregressive Models

In order to compare the results obtained using Markov chains, the VaR was determined applying the classical method by estimating the function approximating the behaviour of a series of returns and the use of the estimated model.

The SARIMA (Seasonal Auto-Regressive Integrated Moving Average) models (p,d,q)×(P, D, Q) **(**Brockwell, Davis, 1996) are used to describe the level of phenomena shaping over time at high frequency of observation, in which autocorrelation and seasonality are used.

$$p(B)P_s(B^s)\nabla_s^d z_t = q(B)Q_s(B^s)\varepsilon_t \, , \tag{3.12}$$

where:

$$p(B) = 1 - \sum_{i=1}^{p} p_i B^i , \qquad P_s(B) = 1 - \sum_{i=1}^{P} P_{si} B^i \, ,$$

$$q(B) = 1 - \sum_{i=1}^{q} q_i B^i , \qquad Q_s(B) = 1 - \sum_{i=1}^{Q} Q_{si} B^i \, ,$$

s – seasonal lag, d – order of series integration,

z_t – empirical values of series,

B – transition operator $B^s z_t = z_{t-s}$,

∇ – differential operator $\nabla^s z_t = z_t - z_{t-s} = (1 - B^s) z_t$,

ε_t – model residuals.

The residuals ε_t of a linear autoregressive model do not meet the conditions of white noise and display a significant ARCH effect, therefore model (3.12) is complemented by a model allowing for heteroscedasticity of variance:

$$\varepsilon_t = \sigma_t \xi_t . \tag{3.13}$$

For the purposes of this work, out of the numerous class of conditional variance models, we selected a model proposed by Glosten, Jagannathan and Runkle (GJR) in 1993:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} (\alpha_i \varepsilon_{t-i}^2 + \gamma_i S_{t-i}^- \varepsilon_{t-i}^2) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 , \tag{3.14}$$

where:

α_0 – the value of unconditional variance of the process ($a_0 \geq 0$),

$\alpha_q, \beta_p > 0$ and the remaining coefficients are non-negative,

$S_{t-i}^- = \begin{cases} 0 & \gamma_i > 0 \\ 1 & \gamma_i < 0 \end{cases}$,

which allows for differences in when impacting variances, past negative values ε_t. Among the models considered for the analysed time series – GARCH, EGARCH, APARCH, IGARCH, FIGARCH, FIEGARCH, FIAPARCH, GJR (Osińska, 2006; Fiszeder, 2009; Trzpiot, 2010) the best fit to empirical data in the sense of the Schwartz criterion (BIC) was the GJR model with Generalized Error Distribution (GED).

Table 3.1. presents the results of the SARIMA-GJR model parameter estimation for linear returns for the gas_base index in the time period 02.01.2014–13.04.2017.

Table 3.1. The SARIMA-GJR model parameter estimation

Parameter	Parameter estimation	Standard error	t-Student statistics	p-value
p(1)	0.7970	0.0502	15.8639	0.0000
q(1)	0.8905	0.0380	23.4505	0.0000
Ps(1)	0.0697	0.0344	2.0229	0.0433
Qs(1)	0.9207	0.0163	56.4169	0.0000
α_0	1.5087	0.7695	1.9610	0.0502
α_1	0.1760	0.0519	3.3900	0.0007
β_1	0.5678	0.1334	4.2550	0.0000
γ	0.2310	0.0773	2.9870	0.0029
G.E.D.(DF)	1.2288	0.0718	17.1200	0.0000

The residuals ξ_t of the obtained model are characterized by absence of autocorrelation, compliance with GED distribution (Figure 3.3) and absence of the ARCH effect (p-value = 0.87).

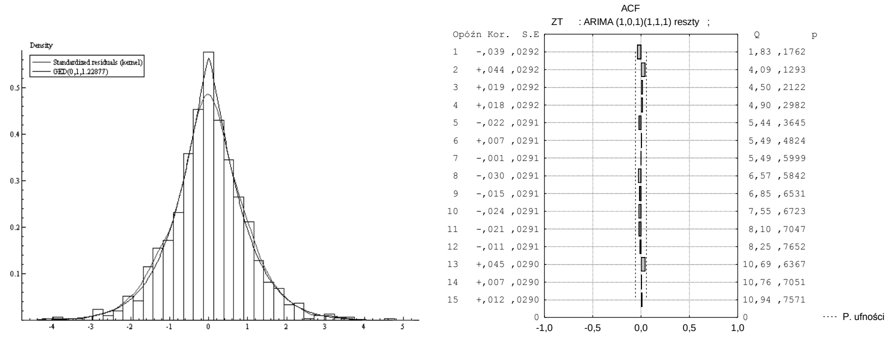

Figure 3.3. Evaluation of SARIMA-GJR model adjustment to empirical series of returns

Figure 3.4. presents a selected part of a time series of returns (z_t) and the estimated $VaR_{0.05}$ for the one-day investment horizon using the theory of stochastic processes (VaR_SGJR).

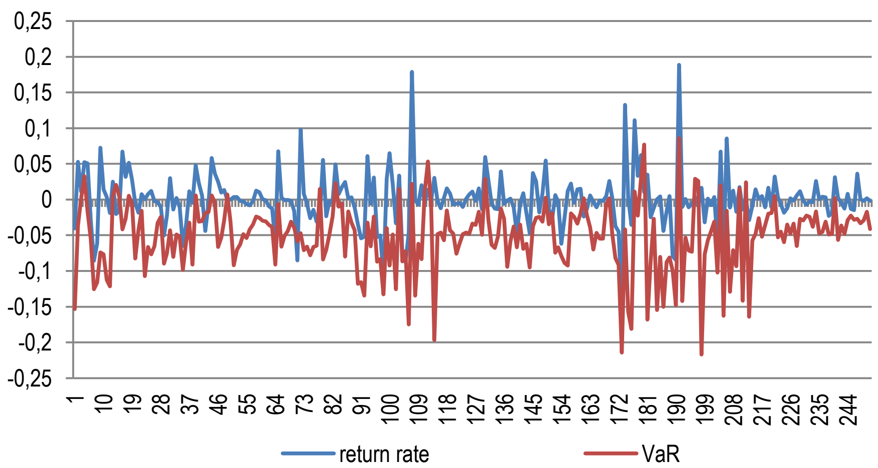

Figure 3.4. The results of VaR estimation for a selected subperiod of 250 observations using SGJR

4. Comparison the Results

In order to compare the obtained results of the VaR estimation we used back testing for the hit function $\left[I_t(\alpha)\right]_{t=1}^{t=T}$

$$I_t(\alpha)=\begin{cases}1 & dla\ \ z_{t+\Delta t}\le -VaR_t(\alpha)\\ 0 & dla\ z_{t+\Delta t}> -VaR_t(\alpha)\end{cases}, \tag{4.1}$$

where:

T – length of time series,

$z_{t+\Delta t}$ – the stochastic process $Z_{t+\Delta t}$.

by means of the following test:

- number of VaR_α exceedances *(Proportion of Failures Test – POF)* (Kupiec, 1995),
- independence of VaR_α exceedances (*Independence Test – IND)* (Christoffersen, 1998).

The test for the number of VaR_α exceedances (*POF*) verifies the following hypothesis:

$$H_0:\quad w_{VaR_\alpha}=\alpha$$

against the alternative hypothesis

$$H_1:\quad w_{VaR_\alpha}\ne\alpha$$

where:

α – the order of VaR_α exceedances

w_{VaR_α} – the participation of VaR_α exceedances in the process of the considered returns.

$\hat{w}_{VaR_\alpha}=\dfrac{K}{T}$ – the participation of VaR_α exceedances (K – the number of exceedances), in the series of the considered returns (T- the length of the series).

Assuming the truth of null hypothesis, the statistics (Kupiec, 1995):

$$LR_{POF} = -2\ln\left\{\frac{(1-\alpha)^{T-K}\alpha^{K}}{\left[\left(1-\frac{K}{T}\right)^{T-K}\right]\left(\frac{K}{T}\right)^{K}}\right\}, \tag{4.2}$$

has an asymptotic distribution χ^2 with one degree of freedom.
The test for independence of VaR_α exceedances (IND) verifies the following hypothesis:

H_0 : VaR_α **exceedances** are independent

against the alternative hypothesis

H_1 : VaR_α **exceedances** are dependent

To verify the null hypothesis, Christoffersen proposed statistics using the Markov chain idea:

$$LR_{IND} = -2\ln\left\{\frac{(1-\overline{w})^{K_{00}+K_{10}}\,\overline{w}^{K_{01}+K_{11}}}{(1-w_{01})^{K_{00}}w_{01}^{K_{01}}(1-w_{11})^{K_{10}}w_{11}^{K_{11}}}\right\}, \tag{4.3}$$

where:

K_{ij} – the number of periods in which $I_t(\alpha) = j$ on condition that $I_{t-1}(\alpha) = i$; $w_{ij} = \frac{K_{ij}}{K_{i0}+K_{i1}}$;

$\overline{w} = \frac{K_{01}+K_{11}}{T} = \frac{K}{T} = \hat{w}_{VaR_\alpha}$,

i, j= 0, 1.

Statistics (3.7) with the assumption of the truth of the null hypothesis has an asymptotic distribution χ^2 with one degree of freedom.

Table 4.1 shows the test results for the estimated VaR. The number of estimated VaRs using Markov chains is equal to the length of the time series (T=1177). For the VaR obtained based on the results of the SGJR model, the loss of the first seven values (T=1170) is related to the seasonal variation of a series of return rates.

For the analysed time series $VaR_{0.05}$ estimation using Markov chains gives an almost expected exceedances participation of 0.0535. Furthermore, the high value of p = 0.0535 of the Kupiec proportion of failures test shows no grounds for rejecting the null hypothesis. For a historical time series of exceedances, there was no single case of day-to-day VaR exceeding.

$VaR_{0.05}$ estimated using the SARIMA-GJR model is slightly underestimated, the participation of exceedances in the examined series is 0.564, not significantly different from the expected (*p*-value = 0.3238 in the Kupiec proportion of failures test). Exceeding the so estimated VaR can be considered as independent (*p*-value = 0.1038 in Christoffersen test).

Table 4.1. Results of $VaR_{0.05}$ back testing

	VaR_M	VaR_SGJR
T	1177	1170
k	63	66
w	0.0535	0.0564
K_{00}	1051	1045
K_{10}	63	59
K_{01}	63	59
K_{11}	0	7
w_{00}	0.9434	0.9466
w_{10}	1.0000	0.8939
w_{01}	0.0566	0.0534
w_{11}	0.0000	0.1061
LR_{POF}	0.3014	0.9736
p-value	0.5830	0.3238
LR_{IND}	x	2.6466
p-value	x	0.1038

Conclusions

The obtained VaR estimation results are far better than VaR estimates based on Monte Carlo simulations without taking into account the dynamics of the observed phenomena and the strong autocorrelation observed during the time series (cf. Ganczarek-Gamrot, 2015). Both methods have a great advantage over the classic approach to Value-at-Risk estimation. Nevertheless, VaR estimated using Markov chains based on the selected empirical series is closer to the correct estimation of loss measured by means of VaR.

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[www 1] www.polpx.pl.

Porównanie wybranych dynamicznych metod estymacji VaR na rynku gazu w Polsce

Zarys treści: W pracy porównano wyniki estymacji wartości zagrożonej VaR oszacowanej przy wykorzystaniu łańcuchów Markowa oraz modeli autoregresyjnych liniowych i nieliniowych. Analizę porównawczą przeprowadzono dla liniowych stóp zwrotu wartości dziennego indeksu gas_base notowanego na Rynku Dnia Następnego (RDN) Towarowej Giełdzie Energii (TGE) w okresie od 2 stycznia 2014 roku do 13 kwietnia 2017 roku. Zgodność i niezależność przekroczeń oszacowanych wartości VaR zweryfikowano testem Kupca oraz Christoffersena.

Słowa kluczowe: VaR, łańcuch Markowa, modele SARIMA, modele GARCH, analiza wsteczna.