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# Cointegration of Interdependencies Among Capital Markets of Chosen Visegrad Countries and Germany

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**Abstract.** Identification of linkages among capital markets is crucial for forming policies that take into account risk associated with international financial markets interdependencies. Thus, the aim of the article is to analyse interdependencies among capital markets of Germany, Poland, Czech Republic and Hungary. The research hypothesis was given as follows: There is a similar course and changes in the interdependencies among capital markets of Germany and the markets of the mentioned countries of the Visegrad Group. In the research a DCC-GARCH model was applied. The model allowed to estimate conditional correlations that indicate strength of the interrelationship among the markets. Then, the cointegration analysis of the conditional correlations was conducted. The proposed econometric procedure allowed to verify the research hypothesis. It confirmed that the capital markets of Germany, Poland, Czech Republic and Hungary are characterised with similar long-term path. Additionally, the research showed that changes in the direction and strength of the interrelationships among the studied markets are determined by the German capital market in the long-term, which is a leader in the region.

**Keywords:** cointegration of interdependencies among capital markets, conditional correlation, DCC-GARCH model, conditional variance

**JEL Classification:** G15, C58

**AMS Classification:** 90C15

## 1 Introduction

Identification of linkages among capital markets and evaluation of their variability over time are crucial for forming guidelines, which can help to “manage” globalized economy [18, 14, 2, 19, 28]. The research in that field is important from the perspective of forming rules and regulations of financial markets that can support their positive influence on the development processes and socio-economic sustainability [3, 5, 31, 7, 8]. It is necessary to propose policies that take into account the risk associated with international financial markets interdependencies. This risk is especially important during the time of global uncertainty, where the instability of one foreign capital market can lead to recession in real sphere in different economies [12, 29, 13, 16, 1, 4] and influence negatively the crucial macro and micro economic spheres such as countries fiscal stability [24, 10,11], situation on labour markets [26, 35, 6, 9, 32, 33] or international competitiveness of national industries [34]. As a result, the purpose of the article is to describe interdependencies among capital markets of Germany, Poland, Czech Republic and Hungary. The analysis was conducted for the years 1997-2015. The research hypothesis was given as follows: There is a similar course and changes in the interdependencies among capital markets of Germany and the markets of the mentioned countries of the Visegrad Group.

In the first step of the research DCC-GARCH model was used in order to estimate conditional correlations that indicate strength of the interrelationship among the analysed markets. In the second stage, the cointegration analysis of the conditional correlations was conducted. The research is continuation of previous studies of the authors in the field [17, 36].

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## 2 Method of the Research

In the research parameters of DCC=GARCH model were estimated [15], which enabled to assess conditional variances and conditional correlations for the pairs of indices for the capital markets of Germany, Poland, Czech Republic and Hungary.

The next step was the cointegration analysis and proposition of VECM model (Vector Error Correction Model), which is a VAR model for cointegrated processes. Let's assume  $p$  rank VAR model with deterministic component  $d_t$ .

$$y_t = d_t + A_1 y_{t-1} + \dots + A_p y_{t-p} + \varepsilon_t \tag{1}$$

where:  $y_t$  is  $N$ -dimensional stochastic process,  $A_i$  for  $i=1\dots p$  are matrixes of parameters  $N \times N$  and  $\varepsilon_t$  is  $N$ -dimensional white noise.

VAR model in the form of differences of the process  $y_t$  can be given with equation 2.

$$\Delta y_t = \mu_t + \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \varepsilon_t \tag{2}$$

where:  $\Pi = \sum_{i=1}^p A_i - I$  and  $\Gamma_i = -\sum_{j=i+1}^p A_j$ .

The rank " $r$ " of a matrix  $\Pi$  determines the interpretation of the model given with equation 2. When the rank  $r$  equals  $N$ , then process  $y_t$  is not integrated [ $I(0)$ ]. When rank  $r$  equals zero, then process  $y_t$  is integrated of order one [ $I(1)$ ], but it is not cointegrated. There is a cointegration when rank  $r$  is between  $0 < r < N$  and matrix  $\Pi$  can be given as  $\alpha\beta'$ . Matrix  $\alpha$  is a matrix of adjustments matrix and  $\beta$  is a cointegrating vector.

In order to test cointegration the existence of  $\Pi$  matrix is determined and then the rank  $r$  of matrix  $\Pi$  is calculated. For this purpose Johansen [21, 22, 23] procedure is applied, where a maximum eigenvalue test is used. The procedure is an iterative process and it is applied until the rejection of the null hypothesis. In the first step of the procedure the null hypothesis that there is no cointegrating vector is adopted (lack of cointegration,  $H_0: r = 0$ ), against the alternative that there is at least one such vector  $H_1: (r > 1)$ . In the case of the absence of evidence that enable to reject the null hypothesis, the following assumptions are made successively: ( $H_0: r \leq 1, H_1: r = 1$ ), ( $H_0: r \leq 2, H_1: r = 2$ ), which in next steps results in successive increasing of  $r$ . The maximum eigenvalue test statistics is characterized with nonstandard distribution, where the critical values can be found in the work of Osterwald-Lenum [27].

## 3 Results of the research

In the research time series for four stock market indexes (BUX, PX 50, WIG and DAX) were used. For each of the indexes a daily rate of return were set for the period: 1 July 1997 to 30 September 2015, which gave a total number of 4592 observations<sup>6</sup>. In the first step of the research parameters of DCC-GARCH model were estimated with application of maximum likelihood method with the t-student conditional distribution. The results are presented in table 1. Additionally for all the indices a constant in the equation conditional mean were estimated, where all the parameters corresponding to the constant were statistically significant at the 5% significance level. For each of the indices the parameters corresponding to the conditional variances and conditional correlations in DCC-GARCH model were also statistically significant.

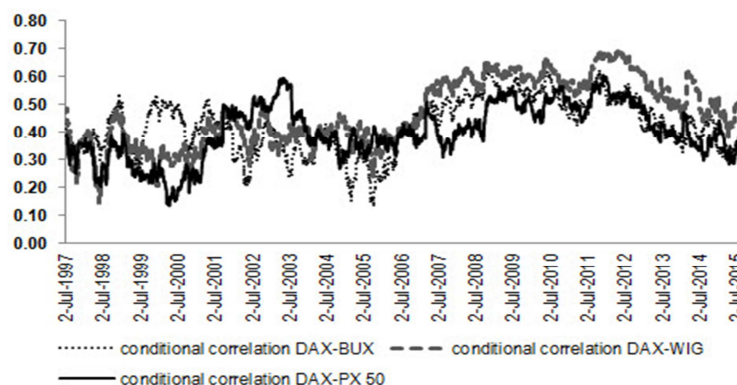
Parameter (stock index)	Estimate	p-value	Parameter (stock index)	Estimate	p-value
const (PX 50)	0.0493	0.0124	const(WIG)	0.0526	0.0013
$\omega_1$ (PX 50)	0.0221	0.0053	$\omega_3$ (WIG)	0.0209	0.0055

<sup>6</sup> The time series for the research were retrieved from: <http://www.finance.yahoo.com> (30.10.2015).

$\alpha_1$ (PX 50)	0.0694	0.0000	$\alpha_3$ (WIG)	0.0771	0.0000
$\beta_1$ (PX 50)	0.9250	0.0000	$\beta_3$ (WIG)	0.9145	0.0000
const (BUX)	0.0580	0.0027	const (DAX)	0.0799	0.0000
$\omega_2$ (BUX)	0.0641	0.0003	$\omega_4$ (DAX)	0.0259	0.0001
$\alpha_2$ (BUX)	0.1058	0.0000	$\alpha_4$ (DAX)	0.0947	0.0000
$\beta_2$ (BUX)	0.8732	0.0000	$\beta_4$ (DAX)	0.8960	0.0000
$a$	0.0084	0.0000	$b$	0.9891	0.0000
$v$	8.4787	0.0000	-	-	-

**Table 1** The results of the estimation of the DCC-GARCH model

The estimation of parameters of DCC-GARCH model allowed to determine the values of conditional correlations for the next pairs of indices. The correlation values for a given pair of indices indicate the strength of the relationship between the two capital markets. Additionally, it gives information on changes of upward or downward trends of these interrelationships over time. In the next stage, based on the aim of the current paper the values of conditional correlations were used to analyze the cointegration.



**Figure 1** The conditional correlation between pairs of chosen markets

Figure 1 shows the conditional correlations between the DAX index and PX 50, WIG, BUX indices. The analysis of correlations shows that the relationships between the capital markets of Germany and the markets of the Czech Republic, Poland and Hungary are quite similar. It can be said that since 2004 the shocks on German capital market have been transferred in a similar way to Czech, Polish and Hungarian markets. This means that the stock valuation on every analyzed capital market is largely dependent on the situation on the other markets.

In the next step an analysis of cointegration of conditional correlations obtained after application of DCC\_GARCH model was conducted. For this purpose a stationarity of time series of conditional correlations was tested with application of Phillips-Perron test [30]. The test results showed that all the time series of conditional correlations are integrated in the order one  $I(1)$ . In the next step a Johansen procedure was carried out, which allowed to test the number of cointegrating relations. Table 2 presents the test results for the number of cointegrating vectors [25]. The results of the max-eigenvalue test indicate the presence of one cointegrating vector.

Hypothesized number of cointegrating vectors	Eigenvalue	Statistic	p-value
None	0.005	26.519	0.007
At most 1	0.002	13.147	0.074
At most 2	0.001	4.880	0.027

**Table 2** Johansen test results

After determination of the number of cointegrating vectors a long-term equation for the model VECM(2) was assessed. The empirical model given with equation 3 was obtained.

$$\rho_{DAX-BUX} = -0.25 + 1.18 \underset{[4.606]}{\rho_{DAX-PX50}} - 1.38 \underset{[-6.561]}{\rho_{DAX-WIG}} + \varepsilon_t \quad (3)$$

where t-statistics are given in square brackets.

The parameters of the long-term equation given with equation 3 are statistically significant. The equation indicates that the conditional correlations between indices DAX and BUX combine a positive relationship with respect to conditional correlation between DAX and PX50, and negative with respect to conditional correlation between DAX and WIG. In the long term, analyzed capital markets form one cointegrating relation with the dominant role of German capital market.

Next the parameters of short-term equations were estimated. Table 3 presents the estimates of parameters and t-statics in brackets for three equations.

Variables	Equations		
	$\Delta\rho_{DAX-BUX}$	$\Delta\rho_{DAX-PX50}$	$\Delta\rho_{DAX-WIG}$
Const	0.0000076 [ 0.06301]	0.0000105 [ 0.08551]	0.0000102 [ 0.08561]
$ECM_{t-1}$	-0.002538 [-2.00532]	-0.001877 [-1.46076]	0.004031 [ 3.21920]
$\Delta\rho_{DAX-BUX,t-1}$	0.004411 [ 0.27181]	0.027074 [ 1.64340]	0.020635 [ 1.28504]
$\Delta\rho_{DAX-BUX,t-2}$	-0.007255 [-0.44694]	-0.019378 [-1.17615]	0.010238 [ 0.63750]
$\Delta\rho_{DAX-PX50,t-1}$	0.002909 [ 0.19442]	-0.013303 [-0.87597]	-0.006694 [-0.45224]
$\Delta\rho_{DAX-PX50,t-2}$	-0.012671 [-0.84720]	0.003501 [ 0.23059]	0.003495 [ 0.23621]
$\Delta\rho_{DAX-WIG,t-1}$	0.028176 [ 1.70311]	-0.018993 [-1.13100]	0.020098 [ 1.22783]
$\Delta\rho_{DAX-WIG,t-2}$	0.016676 [ 1.00766]	0.013382 [ 0.79662]	-0.018205 [-1.11183]

**Table 3** Vector Error Correction Estimates

Negative estimates of parameters for variable  $ECM_{t-1}$  in equations for  $\Delta\rho_{DAX-PX50}$  and  $\Delta\rho_{DAX-BUX}$  were obtained. The obtained sign of estimates means that the pairs of markets are characterized with similar long-term path. On the other hand, in the case of variable  $ECM_{t-1}$  in the equation for  $\Delta\rho_{DAX-WIG}$ , a positive estimate was obtained. It means that the conditional correlation for the pair of indices DAX-WIG is influenced by additional

determinants than the one included in the given model VECM. It can indicate that the German capital market is not the only one that has significant influence Polish capital market in the context of short term deviations.

## 4 Conclusions

The article concentrates on the problem of interrelations among capital markets of chosen Visegrad countries and Germany. The identification of the international relationships among markets should be treated as an important scientific problem. Its adoption is crucial for determination of strategies that can be useful in counteraction of consequences of potential crises. The profound research in that field is a condition for proposing some tools that can be useful in risk management both at micro and macroeconomic level.

The conducted analysis enabled to verify the research hypothesis. It confirmed that the capital markets of Germany, Poland, Czech Republic and Hungary are characterised with similar long-term path. The research showed that changes in the direction and strength of the interrelationships among the capital markets are determined by the German market in the long-term, which can be considered as a leader in the region.

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