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A SPATIAL SAR MODEL IN EVALUATING INFLUENCE OF ENTREPRENEURSHIP AND INVESTMENTS ON UNEMPLOYMENT IN POLAND

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Abstract
The research objective of the article is to analyse the impact of changes in the level of entrepreneurship and business investments on unemployment in Poland with the application of spatial econometrics methodology. A spatial SAR model was used to model unemployment, since this phenomenon exhibits the presence of positive spatial dependence. The research was done for 66 regions at NUTS 3 level for the year 2015.
In order to provide interpretations of the results measures of average impact such as average direct impact, average indirect impact and average induced impact were applied. The obtained results indicate a positive impact of entrepreneurship and investment on decline in unemployment rate and improvement of Poland's socio-economic situation.

Keywords: spatial econometrics, SAR model, spatial dependence, unemployment, entrepreneurship, investments

JEL Classification: *C21, E24*
AMS Classification: 62P20

1 INTRODUCTION

Economic spatial analysis is currently an important direction of development of spatial econometrics (Cliff and Ord, 1981; Klaassen *et al*. 1979; Anselin, 1988; Griffith 1988; Haining, 2005). This is mainly due to its application value, as it can be the basis for effective spatial policy planning at municipal level. Therefore, research effort in this field concentrates on a wide range of phenomena, such as unemployment, quality of human capital, effectiveness of institutions, migration, trade, economic growth and innovation (Müller-Frączek and Pietrzak, 2011, Biczkowski *et al*., 2014; Pietrzak *et al*., 2014; Wilk *et al*., 2013; Pietrzak, 2014a; 2014b; 2014c, Hadaś-Dyduch, 2015; Pietrzak and Łapińska 2015, Balcerzak and Rogalska, 2016; Balcerzak *et al*., 2016; Balcerzak, 2009, 2015, 2016; Hadaś-Dyduch, 2016).
The aim of the article is a spatial analysis of the impact of entrepreneurship and investment on the unemployment rate. The research was done for the Polish sub-regions (NUTS 3). In the analysis, a hypothesis on the impact of growth in the level of entrepreneurship and growth in the level of investment on the decline in unemployment rate at the regional level was proposed. The estimation of parameters of a Spatial Autoregressive Model (SAR) model enabled to confirm the direction of impact of assumed unemployment determinants and to measure their strengths.

2 SPATIAL SAR MODEL AND INTERPRETATIONS OF ITS PARAMETERS

SAR model given with equation from (1) to (4) is a model of linear regression that is additionally enriched by the properties of spatial autoregression (Bivand et al. 2008). The spatial autoregression is added to the model by inclusion of spatial lags of dependent process into the model, which describe the average impact of the process from neighbouring regions on the values of the process in an analysed region. Spatial SAR model can be described as follows:

$$\mathbf{Y} = \rho\mathbf{W}\mathbf{Y} + \sum_{r=0}^{k}\beta_r\mathbf{X}_r + \boldsymbol{\varepsilon} \tag{1}$$

$$(\mathbf{I} - \rho\mathbf{W})\mathbf{Y} = \sum_{r=0}^{k}\beta_r\mathbf{X}_r + \boldsymbol{\varepsilon}, \quad \mathbf{Y} = (\mathbf{I} - \rho\mathbf{W})^{-1}\sum_{r=0}^{k}\beta_r\mathbf{X}_r + (\mathbf{I} - \rho\mathbf{W})^{-1}\boldsymbol{\varepsilon} \tag{2}$$

$$\mathbf{Y} = \mathbf{V}(\mathbf{W})\sum_{r=0}^{k}\beta_r\mathbf{X}_r + \mathbf{V}(\mathbf{W})\boldsymbol{\varepsilon}, \quad \mathbf{V}(\mathbf{W}) = (\mathbf{I} - \rho\mathbf{W})^{-1} \tag{3}$$

$$\mathbf{Y} = \sum_{r=0}^{k}\mathbf{S}_r(\mathbf{W})\mathbf{X}_r + \mathbf{V}(\mathbf{W})\boldsymbol{\varepsilon} \qquad , \ \mathbf{S}_r(\mathbf{W}) = \mathbf{V}(\mathbf{W})\beta_r , \tag{4}$$

where ***Y*** is a vector of dependent process, $\boldsymbol{X}_r$ is a vector of r-explanatory process, k- is a number of explanatory variables ρ is the parameter of the spatial autoregression, **W** is the spatial weight matrix, ***I*** is the matrix of ones, β is r-structural parameter, and ε represents the spatial white noise with a multivariate normal distribution.
In order to interpret the values of parameters of spatial models, three measures of average impact can be used: Average Direct Impact A_D; Average Indirect Impact A_I; Average Induced Impact A_R (LeSage and Pace 2009, Pietrzak, 2013).
Average Direct Impact A_D expresses an average value of the change in the dependent process in any location influenced by the explanatory process X_r of the same location. The measure is calculated as an average of all values $\mathbf{S}_r(\mathbf{W})_{ij}$ when $i = j$ and may be expressed by equation (5):

$$A_D = n^{-1}tr(\mathbf{S}_r(\mathbf{W})), \tag{5}$$

where the *tr* symbol represents the trace of the matrix, n makes a number of observations.
Average Indirect Impact A_I expresses an average change in the dependent process in a freely selected location caused by a change in the explanatory process X_r under the condition that the change in the process X_r occurred in the first-order neighbouring localization (the two locations have common border). The measure can be given with

equation $A_I = n^{-1}tr(\mathbf{W} * \mathbf{S}_r(\mathbf{W})^T)$. (6)

Average Induced Impact A_R expresses an average change in the dependent process in a freely selected location caused by a change in the dependent process X_r , provided that the change in the dependent process X_r occurred in a higher than the first-order neighbouring localization. The locations are neighbours in the sense of the neighbourhood of k-th order, when they are separated by k-borders. The measure can be written with equation (7).

$$A_R = n^{-1}tr(\mathbf{G} * \mathbf{S}_r(\mathbf{W})^T), \ \mathbf{F} = \mathbf{1} - \mathbf{I} - \mathbf{W}_B \tag{7}$$

where **1** is the matrix of ones, **I** is the matrix of ones, $\mathbf{W}_B$ is the first-order neighbourhood binary matrix, **G** s the matrix **F** having row sums normalized.

3 SPATIAL ANALYSIS OF UNEMPLOYMENT IN POLAND

According to the main objective of the article, the spatial SAR model (equation 9) was applied to analyse the determinants of unemployment rate in Poland in the year 2015. The research was conducted for 66 sub-regions (NUTS 3 classification). The number of entities of the economy per capita in 2015 (X_1) and the average level of investment per capita in the years 2012-2015 (X_2) were taken as explanatory processes. Standard neighbourhood matrix, where the neighbourhood is defined basing on the criterion of the common border, is used. In the research, a hypothesis on the positive influence of entrepreneurship and investments on the unemployment rate at the regional level was proposed.

$$\mathbf{Y} = \rho\mathbf{W}\mathbf{Y} + \beta_0 + \beta_1\mathbf{X}_1 + \beta_2\mathbf{X}_2 + \varepsilon . \quad (9)$$

Table 1. The results of the estimation of the *SAR* for unemployment rate in Poland

Parameters	**Estimates**	**p-value**
ρ	0,639	~0,00
β_0	0,102	~0,00
β_1	-0,671	~0,00
β_2	-0,832	~0,00
Properties of the md		
pseudo-R^2	Moran I statistics	p-value
0,672	0,023	0,210

Source: own estimation.

The results of estimation of parameters of the model are presented in Table 1. The autoregression parameter ρ is statistically significant. A positive value of the parameter reaching 0,639 indicates the existence of a strong spatial dependency in the case of unemployment. This means that the change in the situation on the labour market in a given region has a significant impact on the labour markets in other neighbouring regions. This influence is the strongest in the case of first-order neighbouring regions, and it decreases with the increase in the degree of neighbourhood. Also, the parameters β_1 and β_2 are statistically significant, which means that both the changes in the level of entrepreneurship and investment determine the unemployment rate. Negative values of estimations of the parameters β_1 and β_2 allow to verify the hypothesis about the positive influence of these determinants on the decline in the unemployment rate at the regional level. The value of pseudo-R^2 indicates high goodness of fit of the model to empirical data. The value of Moran I statistic indicates that the null hypothesis on absence of spatial autocorrelation of residuals cannot be rejected. This indicates correct properties of random component.

The discussed measures of the average impact can be used as a tool for measuring the impact of explanatory processes on the unemployment rate in the selected regions. The three measures were calculated basing on the estimated values of parameters of the SAR model and the matrix W. The results are presented in Table 2. The influence of X_1 process on the unemployment can be interpreted as follows: an increase of 100 businesses entities per 10,000 inhabitants in any given region will result in an average decrease in the unemployment rate of 0.785% in the same region, the decline in the unemployment rate by an average of 0.143% in the first-order neighbouring regions, and a decline in the unemployment rate by an average of 0.001% in other regions. The impact of the process X_2 can be interpreted by analogy.

Table 2. The values of measures of average impact for SAR model

Process/Measure	**Measure** A_D	**Measure** A_I	**Measure** A_R
Process X_1	-0,785	-0,143	-0,001
Process X_2	-0,927	-0,214	-0,002

Source: own estimation.

The measure of the average impact can be used to assess changes in the labour markets in any selected regions. This can help in forming the guidelines for effective regional policy. The application of the proposed measures to interpret the impact of the explanatory processes will be presented on the example of a hypothetical scenario.
In the scenario, any three neighbouring regions will be considered. The change in the case of explanatory process is assumed only in region 1. An increase of 100 businesses entities per 10,000 inhabitants and an increase in investments of 1,000 PLN per capita is assumed. These changes will not only influence the unemployment rate in region 1, but will also have impact on the neighbouring region 2 and the neighbouring region 3. The average change in the unemployment rate in region 1, which is the result of change of explanatory process in the same region, can be determined by measure A_D. The growth of entrepreneurship in region 1 will result in the fall in the unemployment rate of 0.785%, and as a result of investment growth the unemployment will decrease by 0.927%, which gives a total decline in the unemployment rate of 1,712% in region 1. On the other hand, the average change of the unemployment rate in region 2 and 3, which is the result of change of the explanatory process in the first-order neighboring region (region 1), can be measured with A_I. In the case of the region 2 and the region 3, there will be a total decrease in the unemployment rate of 0.357%, despite the fact that in these regions there has been no change in the level of explanatory processes. The results for scenario are presented in Table 3.

Table 3. The average impact of explanatory variables for a hypothetical scenario

Regions	**Average impact based on the measures**		
	Total effect	**Entrepreneurship**	**Investments**
Region 1	-1,712	-0,785(A_D)	-0,927(A_D)
Region 2	-0,357	-0,143(A_I)	-0,214(A_I)
Region 3	-0,357	-0,143(A_I)	-0,214(A_I)

Source: own estimation.

4 CONCLUSIONS

Not taking spatial interrelations into consideration in the case of many economic phenomena should be considered as an important cognitive error. In the case of research on unemployment, it is very difficult to accept the fact that a change in determinants of the unemployment rate in the selected region will not affect the situation in the neighbouring regions. Such an assumption is made in the case of linear regression model. Only the application of models with spatial autoregression, including SAR models, can allow to take into consideration the existing spatial interdependencies. However, the researcher using the spatial model faces the problem of appropriate interpretation of model parameters. The correct interpretation of the impact of explanatory processes can be made after the application of measures of average impact.
In the case of presented research, the estimation of parameters of SAR model confirmed the existence of strong positive spatial interrelations of unemployment rate. The results confirmed the positive influence of entrepreneurship and investments on the unemployment rate at the regional level.

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