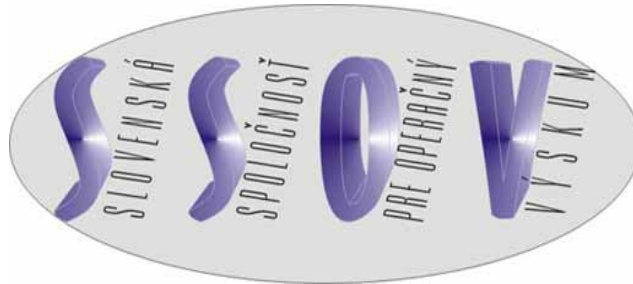**The Slovak Society for Operations Research**
Department of Operations Research and Econometrics
Faculty of Economic Informatics, University of Economics in Bratislava

Proceedings of the International Scientific Conference
QUANTITATIVE METHODS IN ECONOMICS
Multiple Criteria Decision Making XVIII

25th May - 27th May 2016
Vrátna, Slovakia

APPLICATION OF DCC-GARCH MODEL FOR ANALYSIS OF INTERRELATIONS AMONG CAPITAL MARKETS OF POLAND, CZECH REPUBLIC AND GERMANY

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Abstract
The phenomenon of growing capital market linkages is a significant exogenous factor affecting the effectiveness of national economic policies and risk management processes in enterprises. As a result the identification of interdependencies among capital markets is important both from the macro and microeconomic perspective. In this context the main aim of this article is to examine the relations among capital markets of Poland, Czech Republic and Germany. In the research DCC-GARCH model with the t-student conditional distribution was applied. The analysis was conducted for the years 1997-2015. The research findings confirmed significant interdependencies among analysed capital markets, which were evaluated here by conditional correlations.

Keywords: *interdependences among capital markets, conditional variance and correlations, DCC-GARCH model*

JEL Classification: G15, C58
AMS Classification: 62P20

INTRODUCTION

The growing interdependencies among capital markets are currently considered as one of the most important forces responsible for globalisation of the world economy. As a result, these interrelations have been a subject of significant empirical investigations in recent years (Forbes and Rigobon, 2002; Baur, 2003; Corsetti et al., 2005; Billio and Caporin, 2010; Fałdziński and Pietrzak, 2015; Heryán and Ziegelbauer, 2016). The interdependencies of markets can be especially important during the time of international instability. They can provide transmission channels for negative impulses that destabilise the economies in the real sphere, which can be difficult to control at national level (Balcerzak, 2009a, 2009b). As a result, identification of the interdependencies among capital markets and analysis of their strength in time should be considered as a tool for managing risk related to financisation of economy. In this context the main purpose of this article is to examine interrelations between capital markets of Poland, Czech Republic and Germany. Polish and Czech stock exchanges can be treated as relatively young neighbouring markets. On the other hand, quite close geographically German capital market is mature and one of the biggest markets in Europe, thus it should have significant influence on Polish and Czech stock exchanges.

In the research DCC-GARCH model was applied as it enables to analyse the interrelations among markets by estimating the time-varying conditional correlation for given pairs of markets. The research was conducted for the years 1997-2015, which is a period long enough for drawing conclusions on the interrelations among chosen markets.

DCC-GARCH MODEL SPECIFICATION

Originally GARCH models enabled to model conditional variance for individual assets or indices. However, some early applications of the models showed that the capital markets should not be analysed separately but also interdependences between them should be taken into account. That problem was solved after introduction of DCC-GARCH models that enable to analyse interdependence among markets by estimating the time-varying conditional correlation (Engle, 2002). After adding ARMA (p, q) model to DCC-GARCH model, it is possible to catch the presence of autocorrelation of returns or autocorrelation of random disturbance. Model ARMA-DCC-GARCH can be given as follows:

$$(1 - \Phi_1 B - ... - \Phi_p B^p)\mathbf{Y_t} = \mathbf{c} + (1 + \Theta_1 B + ... + \Theta_q B^q)\boldsymbol{\eta_t}, \; \boldsymbol{\eta_t} \,|\, F_{t-1} \sim t(0, \mathbf{D_t R_t D_t}, v) \tag{1}$$

$$\mathbf{D_t^2} = diag\{\mathbf{H_t}\}, \; \mathbf{H_t} = \mathbf{V_{t-1}}(\boldsymbol{\eta_t}) \tag{2}$$

$$H_{i,t} = \omega_i + \alpha_i \eta_{i,t-1}^2 + \beta_i H_{i,t-1} \tag{3}$$

$$\boldsymbol{\varepsilon_t} = \mathbf{D_t^{-1}} \boldsymbol{\eta_t} \tag{4}$$

$$\mathbf{R_t} = diag\{\mathbf{Q_t}\}^{-1/2} \mathbf{Q_t} \, diag\{\mathbf{Q_t}\}^{-1/2} \tag{5}$$

$$\mathbf{Q_t} = \boldsymbol{\Omega} + \alpha \boldsymbol{\varepsilon_{t-1}} \boldsymbol{\varepsilon_{t-1}'} + \beta \mathbf{Q_{t-1}}, \; \boldsymbol{\Omega} = \mathbf{\bar{R}}(1 - \alpha - \beta), \tag{6}$$

where:

$\mathbf{Y_t}$ - the multivariate process of returns, $\boldsymbol{\mu_t}$ - the vector of conditional means of returns, $H_{i,t}$ - the conditional variance for i-th returns, where $i = 1, ..., N$,

$\mathbf{R_t}$ - the time-varying conditional correlation matrix,

$\mathbf{V_{t-1}}(\boldsymbol{\eta_t})$ - the conditional covariance matrix of the residuals $\boldsymbol{\eta_t}$,

c – the vector of constant,

$\boldsymbol{\Phi, \Theta}$ - the parameters of the ARMA(p, q)

$\omega_i, \alpha_i, \beta_i$ - the parameters of the conditional variance equation, where $i = 1, ..., N$,

α, β - the parameters of the conditional correlation equation,

$v > 2$ - the number of degrees of freedom in the t-distribution,

$\mathbf{\bar{R}} = \dfrac{1}{T} \sum_{i=1}^{T} \boldsymbol{\varepsilon_t} \boldsymbol{\varepsilon_t'}$ - the unconditional covariance matrix.

The estimation of parameters of DCC-GARCH model can be carried out using the maximum likelihood method. The two step estimation method can be applied here (Engle, 2002; 2009). In the first step the parameters of the conditional variance equations and means are estimated. In the second step the parameters of the conditional correlation equation are estimated.

APPLICATION OF THE MODEL

In the research interdependencies among capital markets of Germany, Poland and Czech Republic were analysed. Thus, time series for three indices were used (DAX, WIG20 and PX 50). For this purpose a daily observations from 1 July 1997 to 30 September 2015 were taken, which gives T = 4592 observations. The data were obtained from the service http://www.finance.yahoo.com. In the study logarithmic returns $r_t = 100(\ln(P_t) - \ln(P_{t-1}))$ were used. In the estimation of parameters of DCC-GARCH model the maximum likelihood method with a fat-tailed t-distribution was applied.

Table 1 presents the results of estimation of parameters of ARMA-DCC-GARCH model. For all market indices autoregression parameter ϕ_i for the conditional mean equation were statistically significant at the significance level of 5%. All the parameters for conditional variances and correlations were also statistically significant. Parameter estimate at the level 9.7505 was obtained for the parameter v, which indicates the correct adjustment of the t - distribution to the data.

Table 1. The results of estimation of parameters of ARMA-DCC-GARCH model

Parameter (stock index)	Estimate	Std. Error	p-value
The conditional means and variances equations			
Germany			
c_1 (DAX)	0.0243	0.0168	0.0147
ϕ_1 (DAX)	0.0137	0.0150	0.0360
ω_1 (DAX)	0.0328	0.0058	0.0000
α_1 (DAX)	0.0746	0.0095	0.0000
β_1 (DAX)	0.9131	0.0088	0.0000
Poland			
c_2 (WIG 20)	0.0355	0.0166	0.0329
ϕ_2 (WIG 20)	0.0204	0.0069	0.0034
ω_2 (WIG 20)	0.0640	0.0111	0.0000
α_2 (WIG 20)	0.9286	0.0118	0.0000
β_2 (WIG 20)	0.1796	0.0463	0.0001
Czech Republic			
c_3 (PX 50)	0.0127	0.0191	0.0252
ϕ_3 (PX 50)	0.0417	0.0159	0.0000
ω_3 (PX 50)	0.0167	0.0053	0.0000
α_3 (PX 50)	0.0704	0.0116	0.0000
β_3 (PX 50)	0.9366	0.0113	0.0000
The conditional correlation equation			
α	0.0060	0.0011	0.0000
β	0.9937	0.0013	0.0000
v	9.7505	0.6594	0.0000

Source: own estimation based on data from http://www.finance.yahoo.com.

Conducted estimation of ARMA-DCC-GARCH model parameters allowed determining the values of the conditional correlation for the pairs of indices (Figure 1). The correlation values for a particular pair of indices indicate the strength of the relationship between the two capital markets. It also shows changes of upward or downward trends of these interrelationships over time.
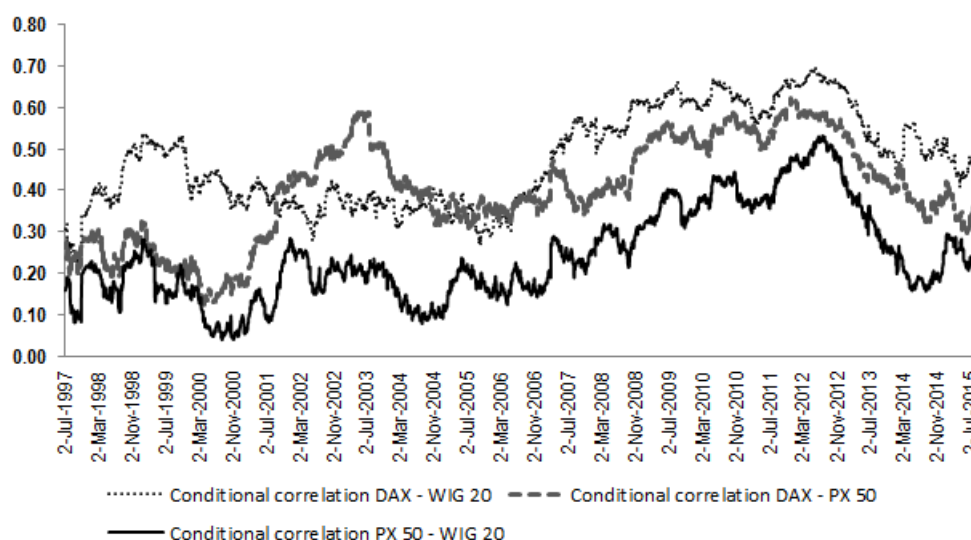
Figure 1. The conditional correlation between the DAX index, the WIG 20 and the PX 50
Source: own estimation based on data from: http://www.finance.yahoo.com.

Figure 1 shows the conditional correlations between the DAX, PX 50 and WIG20 indices. The findings allow making the following conclusions. Firstly, in the case of Czech Republic and Poland, the interdependences among these markets and German capital market are at a medium level. On the other hand, the interrelations between the Polish and Czech markets are much weaker. Next, it should be emphasized that in the period 2004-2015 trends in changes in the value of the conditional correlation (declines and increases) are similar between the capital market of Germany and Polish and Czech markets. It can be said that the shocks occurring on the capital market in Germany have been transferred in a similar way to Czech and Polish markets since 2004. This finding confirms the results of previous financial studies that the situation on any stock exchange is to some extent dependent on the situation on the other markets.

CONCLUSIONS

The article concentrates on the phenomena of growing interrelations among capital markets. Their strength is constantly increasing due to intensification of globalization process. The identification of the interdependencies among capital markets has been considered as an important scientific problem in recent years. It can be useful in developing of tools and regulations that can improve functioning of capital markets. The adoption of such tools can be helpful in determination of strategies during potential crisis.

The main aim of the article was to examine the interdependencies among the capital markets of Germany, Poland and Czech Republic. The research findings allow to assess the interrelations among these markets and enable to evaluate changes in these interrelationships over time. To some extent these interdependencies were shape differently depending on the selected pair of markets. This means that in spite of the interdependence among analysed indices, the markets in question cannot be treated as a homogeneous area. Thus, the single markets function in a specific way, and react differently to similar external shocks.

However, it should be noted that for all pairs of indices an increase in the value of the conditional correlation have been noticeable since 2004. This may be a result of the accession of Poland and Czech Republic to the European Union, which contributed significantly to the economic growth of these countries influencing also the dynamics of their capital markets. Then, in the years 2007-2008 there was a further increase in interdependence among the

markets. The increase in the interrelations among the analysed markets in this period could be the result of the contagion effect that occurred in the markets due to the global financial crisis. The increase in the level of interdependence lasted until the year 2012. In the subsequent period 2013-2015 a downward trend in the interdependencies can be observed. This may be a sign of a slow return to long term equilibrium on the capital markets, thus to the period of greater economic stability.

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