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MEREOLOGY THEN AND NOW

Abstract. This paper offers a critical reconstruction of the motivations that led to the development of mereology as we know it today, along with a brief description of some questions that define current research in the field.

Keywords: mereology; parthood; formal ontology; foundations of mathematics

1. Introduction

Understood as a general theory of parts and wholes, mereology has a long history that can be traced back to the early days of philosophy. As a formal theory of the part-whole relation – or rather, as a theory of the relations of part to whole and of part to part within a whole – it is relatively recent and came to us mainly through the writings of Edmund Husserl and Stanisław Leśniewski. The former were part of a larger project aimed at the development of a general framework for formal ontology; the latter were inspired by a desire to provide a nominalistically acceptable alternative to set theory as a foundation for mathematics. (The name itself, ‘mereology’ – after the Greek word ‘μερος’, ‘part’ – was coined by Leśniewski [31].) As it turns out, both sorts of motivation failed to quite live up to expectations. Yet mereology survived as a theory in its own right and continued to flourish, often in unexpected ways. Indeed, it is not an exaggeration to say that today mereology is a central and powerful area of research in philosophy and philosophical logic. It may be helpful, therefore, to take stock and reconsider its origins. For

it is precisely from their relative failure that many open questions in current research find their source.

2. Mereology as formal ontology

Husserl's conception of mereology as a piece of formal ontology finds its fullest formulation in the third of his *Logical Investigations* [27]. There, Husserl was interested in the development of

the *pure (a priori) theory of objects as such*, in which we deal with ideas pertinent to the *category of object* [...] as well as the *a priori* truths which relate to these. [27, p. 435]

Husserl mentioned several other ideas besides Part and Whole, including Genus and Species, Subject and Quality, Relation and Collection, Unity, Number, Series, Ordinal Number, Magnitude, etc. Yet the bulk of the Investigation is devoted to the first idea and the title itself (“On the Theory of Wholes and Parts”) is indicative of the centrality of the part-whole relation in his project.

The very notion of an “object as such” is, of course, heavily laden with philosophical meaning, as is Husserl's notion of an *a priori* truth. For our present purposes, however, the central idea can be put rather simply as follows. Don't think of ontology in Quinean terms, i.e., as a theory aimed at drawing up an inventory of the world, a catalogue of those entities that must exist in order for our best theories about the world to be true [46]. Rather, think of ontology in the old sense of a theory of being *qua* being (Aristotle), or perhaps of the possible *qua* possible (Wolff). In this sense, the task of ontology is not to find out what there is; rather, its task is to lay bare the formal structure of what there is *no matter what it is*. Regardless of whether our domain of quantification includes objects along with events, abstract entities along with concrete ones, and so on, it must exhibit some general features and obey some general laws, and the task of ontology – understood formally – is to figure out such features and laws. For instance, it would pertain to the task of formal ontology to assert that every entity, no matter what it is, is governed by certain laws concerning identity, such as reflexivity, symmetry, or transitivity:

- (1) $x = x$
- (2) if $x = y$, then $y = x$
- (3) if $x = y$ and $y = z$, then $x = z$.

The truth of these laws does not depend on what (sorts of) entities are assigned to the individual variables ‘ x ’, ‘ y ’, and ‘ z ’, exactly as the truth of the following laws concerning equivalence does not depend on what propositions are assigned to the sentential variables ‘ p ’, ‘ q ’, and ‘ r ’:

- (1′) $p \equiv p$
- (2′) if $p \equiv q$, then $q \equiv p$
- (3′) if $p \equiv q$ and $q \equiv r$, then $p \equiv r$.

Whereas the latter laws pertain to formal logic, precisely insofar as the relevant variables range over propositions, i.e., statements about the world (no matter what they say), the former would pertain to ontology insofar as their variables range over things in the world (no matter what they are). But both sorts of truth are meant to possess the same sort of generality and topic-neutrality. Both are meant to hold as a matter of necessity and should be discovered *a priori*.

Now, it is clear that this conception of ontology faces two sorts of difficulty.¹ The first concerns the bounds of the theory itself: what sorts of relation may be said to apply to anything that might conceivably exist, regardless of its nature? Identity appears to be an obvious candidate; the very idea that we should only countenance entities for which we have clear identity criteria, as per Quine’s famous *dictum* [47, p. 20], is indicative of the putative generality of this relation. But what other relations should be included in the domain of the theory? In particular, is the part-whole relation genuinely formal in this sense? Obviously Husserl thought so, and many a mereologist have been working under the same hypothesis. Parthood seems to apply to entities as different as material bodies (the handle is part of the mug), events (the first act is part of the play), geometric entities (the point is part of the line), etc. Even abstract entities, such as sets, appear to be amenable to mereological treatment, witness David Lewis’s account in [34]. Yet the general applicability of the part-whole relation is controversial. Just to mention one prominent example, Lewis himself famously argued that entities such as universals *cannot* be structured mereologically, short of unintelligibility:

Each methane molecule has not one hydrogen atom but four. So if the structural universal *methane* is to be an isomorph of the molecules that are its instances, it must have the universal *hydrogen* as a part not just

¹ Both sorts of difficulty have a parallel in the context of formal logic (see [65]).

once, but four times over. [...] But what can it mean for something to have a part four times over?² [33, p. 34]

The second sort of difficulty concerns, not the bounds of formal ontology, but its contents. Surely not every general thesis concerning identity qualifies as a *formal* law on a par with reflexivity, symmetry, and transitivity. Leibniz's principle of the Identity of Indiscernibles, for example, is arguably a substantive thesis that can hardly be claimed to hold *a priori* (see e.g. [5]).³ Likewise, not every mereological thesis would qualify as formal in the relevant sense, even if parthood were granted the status of a formal ontological relation. Consider, for instance, the question of whether there are mereological *atoms* (i.e., entities with no proper parts), or of whether everything is ultimately *composed of* atoms. Clearly any answer to such questions would amount to a substantive metaphysical thesis that goes beyond a "pure theory of objects as such". The same could be said of other theses that are often discussed in connection with the part-whole relation, such as the principle of *extensionality* (according to which no two composite wholes can have the same proper parts) or the principle of *unrestricted composition* (according to which any group of objects compose a whole).⁴ What, then, are the laws of the "pure theory"? What principles define the core of mereology understood as a *formal* theory of parts and wholes? Husserl himself went some way towards answering such question⁵, and most theories that followed have been developed in the same spirit. Specifically, let ' \sqsubseteq ' represent the (proper or improper) parthood relation and let ' \sqsubset ' stand for its proper restriction. Then the general idea has been that \sqsubset is essentially a strict partial ordering with the property that every proper part is always supplemented by another, disjoint part:

² For the record, Lewis takes this to be a *reductio ad absurdum* of the very idea that there are structural universals; but, of course, the friend of such entities may take it instead as a *reductio* of the idea that their mode of composition is mereological (see e.g. [2]).

³ Indeed, even the general laws mentioned above have sometimes been called into question. For instance, it has been argued that the loss of individuality in the quantum realm results in a violation of the law of reflexivity [18], or that the transitivity of identity is violated by so-called vague objects [45].

⁴ For a review of the philosophical issues raised by these principles, see [66], esp. §3.2 and §4.5.

⁵ The principles put forward in [27] are "only meant to count as mere indications" to be further worked out (p. 484). For detailed developments see e.g. [50, 37, 16, 7].

- (4) if $x \sqsubset y$, then $y \not\sqsubset x$
- (5) if $x \sqsubset y$ and $y \sqsubset z$, then $x \sqsubset z$
- (6) if $x \sqsubset y$, then there is some $z \sqsubseteq y$ disjoint from x , i.e. such that $w \not\sqsubseteq z$ for all $w \sqsubseteq x$.

These principles seem general enough, to the point of being viewed by some authors as “constitutive of the meaning of ‘part’” [51, p. 11]. Yet, again, their purely formal status has been questioned on various grounds. The last principle, for instance, fails on those theories of material constitution according to which a material object contains the matter that constitutes it as a proper part even though there is nothing to make up for the difference, or it fails on those region-based theories of space according to which a topologically closed region includes its open interior in spite of there being no boundary element to distinguish them. (On these and other examples, see [66, §3.1].) One may be inclined to dismiss such theories as unintelligible precisely because they violate (6); but one might as well go the other way around and regard their independent plausibility as evidence *against* the generality of (6). Ditto for those metaphysical theories that run afoul of either (4) or (5). (See [66, §2.1].) To the extent that such theories cannot be ruled out *a priori*, the principles in question cannot be regarded as metaphysically neutral either, exactly as with the controversial principles of extensionality and unrestricted composition, and one may be led to conclude that there is

no reason to assume that any useful core mereology [...] functions as a common basis for *all* plausible metaphysical theories. [15, p. 246]

Taken together, then, these two sorts of difficulty represent a serious challenge to the idea that mereology can form a genuine piece of formal ontology. The part-whole relation may apply to a very broad range of domains, and within most of these domains it may behave in accordance to such principles as (4)–(6). But there is a growing consensus that this is the best one can say, and that mereology is best understood as a theory — or a plurality of theories — whose fundamental truths do not reflect the properties of the part-whole relation itself but the nature of the entities to which it applies. This is obviously not what Husserl had in mind. Yet precisely here, in the apparent failure of the Husserlian conception of part-whole as a formal ontological relation, lies the richness of much contemporary work in mereology.

On the one hand, the first difficulty has led to a significant amount of research devoted to determining more clearly the range of domains

that are amenable to mereological treatment. Specifically with respect to the case of structural universals mentioned above, Lewis’s misgivings have led most authors to concede the limits of mereology (as in [2]). But there have been also several attempts to develop stronger mereological theories that take the challenge at face value, showing how to account for the idea that something can be part of something else “many times over” (see e.g. [36, 4]). The contributions by Aaron Cotnoir and Peter Forrester to this special issue are indicative of this line of research.

On the other hand, the second difficulty has resulted in the development of a number of “non-standard” mereologies that have significantly changed the map of the field. Not only do we have a better understanding of the variety of theories that lie between the core determined by (4)–(6) and the richer systems obtained by adding suitable versions of the extensionality and unrestricted composition principles (see [51, 66]). We also have a better understanding of what sort of mereology results when one or more of the core principles are dropped. Some of this work has been conducted mainly in philosophical terms (see [10]), but see e.g. [42] for a systematic study of mereologies in which the transitivity axiom (5) may fail and [13] for a thorough investigation of “non-wellfounded” mereologies obtained by dropping the asymmetry postulate (4). None of this would have been possible if mereology were constrained by the ideal of a single, pure ontological theory. Yet it is precisely by reflecting on the tenability of such an idealized approach that these developments have become possible.

3. Mereology as a foundation for mathematics

The other main motivation for the development of mereology, historically, comes from the philosophy of mathematics and can be traced back to Stanisław Leśniewski’s pioneering work, especially his *Foundations of the general theory of sets* [30] and his series of articles “On the foundations of mathematics” [31]. Here the incentive was the persuasion that set theory had been conceived in sin — the sin of intellectual carelessness embodied in founding all of mathematics on such abstract entities as Cantorian sets⁶ — and that the theory of the part-whole relation could provide a more solid foundation. In Leśniewski’s own words:

⁶ Although Leśniewski himself wrote that his conception “is based on a strong scientific tradition represented more or less consistently by numerous scientists old

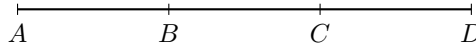


Figure 1. A line segment composed of smaller line segments.

Scenting in the ‘classes’ of Whitehead and Russell and in the ‘extensions of concepts’ of Frege the aroma of mythical specimen from a rich gallery of invented objects, I am unable to rid myself of an inclination to sympathize ‘on credit’ with the authors’ doubts as to whether such ‘classes’ do exist in the world. [31, p. 224]

For Leśniewski, a sworn nominalist, classes (sets) were not abstract entities but objects of a very concrete nature, and “the conception of ‘empty classes’ [is] a ‘mythological’ conception” [31, p. 202]. More specifically, let us write ‘ $\llbracket z \mid z \text{ is a } \varphi\text{-er} \rrbracket$ ’ for the class of all φ -ers (where φ is any condition), and similarly for $\llbracket z_0, \dots, z_n \rrbracket$. Then Leśniewski’s views can be summarized as follows (from [31, pp. 202–203]):

- (a) The class $\llbracket z \mid z \text{ is a } \varphi\text{-er} \rrbracket$ exists only if there is at least one φ -er.
- (b) One and the same object can be identical with classes of different objects. For example, with reference to Figure 1, we have both $AD = \llbracket AB, BD \rrbracket$ and $AD = \llbracket AB, BC, CD \rrbracket$ (hence $\llbracket AB, BD \rrbracket = \llbracket AB, BC, CD \rrbracket$).
- (c) If x is the only φ -er, then $x = \llbracket z \mid z \text{ is a } \varphi\text{-er} \rrbracket = \llbracket x \rrbracket$.
- (d) If y is a class, then $x \sqsubseteq y$ iff, for some φ , x is a φ -er and $y = \llbracket z \mid z \text{ is a } \varphi\text{-er} \rrbracket$.

It is immediately seen that (a)–(c) are in stark contrast with the basic laws of Cantorian set theory, which violate each of these conditions. In particular, (a) is violated precisely because Cantorian set theory acknowledges the empty set, and (c) because of the Axiom of Pairing, which allows for the formation of singletons that are distinct from their elements. As for (b), we have $AD \neq \{AB, BD\}$ and $AD \neq \{AB, BC, CD\}$, since treated as a Cantorian set the segment AD is normally construed as a set of points (not of segments). These three elements of disagreement are the hallmark of Leśniewski’s nominalism, and will be central also in the work of later nominalists (beginning with Nelson Goodman [20]). However, we must delve a bit further to see that the relation \sqsubseteq in (d) is different from the membership relation \in of standard set theory. Let x be

and new, but in particular by Georg Cantor” [31, p. 207], today we know that it was indeed quite different from the conception developed by the German set theorist.

any given object. Clearly, x is the only satisfier of the condition ... *is identical with* x . Thus, by (c) $x = \llbracket y \mid y = x \rrbracket = \llbracket x \rrbracket$. Further, by (d) we have that $x \sqsubseteq \llbracket x \rrbracket$. Hence $x \sqsubseteq x$. It follows that every set is an element of itself (in the sense of \sqsubseteq), and this is not true of Cantorian sets (for which we always have $x \notin x$).⁷ In (b) it can easily be seen that by Leśniewski understood *being an element of* in terms of the mereological notion *being part of*, and in (c) we can see that this notion corresponds to the reflexive relation of (improper) parthood.

Now, both ontological and technical assumptions led Leśniewski to the conclusion that the foundations of mathematics must not only be reconstructed, but fully constructed anew, taking either \sqsubset or \sqsubseteq as the fundamental primitive instead of \in .⁸ The main step of the project was to find a mereological definition of “class” that would fit theses (a)–(c) above. This is the origin of the notion of mereological “sum” that is so characteristic of mereology as we know it today. In terms of the basic language introduced so far, it can be put as follows:⁹

(e) x is a sum of the φ -ers $\longleftrightarrow_{\text{df}}$ every φ -er is part of x and every part of x has a part in common with some φ -er.

Given (e), the bulk of Leśniewski’s system is essentially the theory defined by taking as axioms the three theses (4)–(6) of the previous Section (or the equivalent variant obtained by taking \sqsubseteq to be a partial order) together with the principle of Unrestricted Composition, which can now be formulated as follows:

(7) If there is at least one φ -er, then there exists an x that is a sum of all φ -ers.

From (5), (6) and (7) we obtain the uniqueness of mereological sums (see e.g. [38]):

⁷ Whether there is any set x for which $x \in x$ is a matter of axioms. In Zermelo-Fraenkel set theory (and similar systems) this possibility is ruled out by the so-called *Axiom of Foundation (Regularity)*; by contrast, in Aczel’s non-wellfounded theory [1] there are sets that are elements of themselves just as there are sets that are not. In Leśniewski’s system, however, *every* set is its own element.

⁸ Leśniewski’s very first axiomatization of mereology in 1916 was based on \sqsubset , with \sqsubseteq defined in the obvious way; see [30, pp. 131–132]. Already in 1920, however, he was working on alternative axiomatizations based on \sqsubseteq ; see [31, ch. VII]. It should be noted that throughout his writings Leśniewski used ‘part’ (*część*) for the relation \sqsubset of proper part; his term for \sqsubseteq was ‘ingredient’ (*ingredjens*).

⁹ For more on the mereological definition of class, see [22].

(8) If x and y are sums of φ -ers, then $x = y$.

Thus, in light of (7) and (8), sums always exist — providing there is at least one object to be summed — and are unique, and one can finally set:

(f) $\llbracket z \mid z \text{ is a } \varphi\text{-er} \rrbracket =_{\text{df}}$ the unique x such that x is a sum of the φ -ers.

This is the system known as *Classical Mereology*¹⁰, and it is definitionally equivalent to the nominalistic *Calculus of Individuals* developed independently by Henry Leonard and Nelson Goodman [28].¹¹ Among other things, it implies the extensionality principle for parthood:

(9) If there is a z such that $z \sqsubset x$, and if, for every z , $z \sqsubset x$ iff $z \sqsubset y$, then $x = y$.

It is worth mentioning that, besides his nominalistic motivations, Leśniewski's project was also driven by his desire to solve Russell's antinomy, which he addressed as early as in 1914 [29]. Since, as shown above, his conception of sets led to the conclusion that every set coincides with its own singleton (in the sense of \sqsubseteq), it is clear how he thought of achieving this result, for on his theory one cannot even consider the set of all those sets that are not elements of themselves.¹²

As far as his general program is concerned, however, we can now say that it failed to live up to expectations. Tarski [55] was the first to point out that Leśniewski's axioms determine a class of structures that strongly resemble complete Boolean lattices. More precisely, given any complete Boolean algebra, we can turn it into a model of classical mereology by, *mutatis mutandis*, deleting the zero element. And *vice versa*, any model of mereology can be turned into a complete Boolean lattice upon adding an element to serve as the zero of the structure.¹³ This is a nice result, but of course it means that classical mereology is far too weak to allow for a reconstruction of mathematics. Yet, again, the limits of Leśniewski's original program need not by themselves amount to

¹⁰ Strictly speaking, the system that best deserves this title is the equivalent one determined by (4), (5), (7) and (8). Historically, this is the very first axiomatization of mereology from [31, 1928].

¹¹ On the relationship between Leśniewski's system and Leonard and Goodman's (see [38, pp. 109–114, 127–129] and [51], esp. §2.8).

¹² For more on Leśniewski's approach to Russell's antinomy, see [39, 52, 53, 61].

¹³ For details see e.g. [40, pp. 230–233] and [44]. Specifically with respect to Leśniewski's formulation, see [9].

the failure of any attempt to understand set theory in mereological terms and to see how far one can go using \sqsubseteq instead of \in . And while it is true that not much work has been done in this direction, the publication of Lewis's *Parts of Classes* [34] has prompted renewed interest and opened new prospects of research (see e.g. [17, 6]). The contribution by Joel David Hamkins and Makoto Kikuchi in *Part II* of the present collection, in which they investigate the consequences of taking set inclusion, \sqsubseteq , as the primary fundamental set-theoretic notion in place of set membership, \in , is indicative of how much work can still be done in this direction.

4. Beyond foundations

Despite its relative failure *vis à vis* its original motivations, then, mereology has survived both as a subject of philosophical and logical scrutiny and as a tool in its own right for applications in ontology and the foundations of mathematics. Indeed, the range of areas in which mereology has either been examined or applied is so broad and constantly growing that it is impossible to offer a succinct survey. Here we confine ourselves to a brief list of those aspects that are currently in the foreground, with special reference to the topics covered by the contributions to the present collection (the second part of which will appear in the next volume of *Logic and Logical Philosophy*).

4.1. Metamereology

A major area of current research is the study of mereology from a metamathematical perspective, also known as metamereology (from [38]). This is an area that has attracted the attention of logicians and mathematicians since Tarski's seminal work cited above. Recently, however, the variety of topics addressed under this heading has grown far beyond the narrow scope of mereology's relationship to Boolean algebras and include both model-theoretic and proof-theoretic investigations. Major examples include:

- alternative definitions of mereological sum ([22, 26, 38, 51]) and their relations to the mathematical concept of least upper bound ([23]);
- alternative axiomatizations of classical mereology and weaker theories ([38, 41]), including Gentzen-style proof-systems ([35]);

- decidability of mereological theories ([56, 57, 58, 59]);
- atomistic vs. non-atomistic mereologies ([11, 49, 60]).

Some of these topics are further pursued in this collection. In particular, Hsing-Chien Tsai’s contribution to *Part I*, “Notes on models of first-order mereological theories”, addresses the central problem of definability of subsets of mereological models and its bearing on the question of atomicity and on issues pertaining to the decidability of mereological theories, while Andrzej Pietruszczak’s essay, “Classical mereology is not elementarily axiomatizable”, contains a proof of the fact that the class of all structures satisfying the axioms of classical mereology is not elementarily axiomatizable in the language of the theory (with ‘ \sqsubseteq ’ as a primitive). In *Part II*, Karl-Georg Niebergall’s contribution, “Mereology and infinity”, investigates to what extent finitude and infinity may be captured by mereology, and Paolo Maffezioli’s, “Sequents for non-wellfounded mereology”, explores the proof theory of mereologies allowing for parthood circularity, providing a cut-free sequent calculus that is equivalent to the more familiar axiomatic system of [13]. In addition, *Part II* also contains the above-mentioned essay by Joel David Hamkins and Makoto Kikuchi on “Set-theoretic mereology”.

4.2. Mereogeometry and mereotopology

A second, major area of research involves applications of mereology to geometry and topology. It has long been thought that mereology offers a natural apparatus for modeling the structure of space and the logic of spatial reasoning (see [8, 64]). In recent years, this has proved particularly effective in connection with so called point-free theories of space, i.e., theories that regard the continuum as a connected system of extended regions rather than sizeless points. Such theories stem from the general idea that all talk of points and boundary-like entities must be seen as involving some sort of abstraction — an idea that can be found already in the medieval and modern debates on anti-indivisibilism but that made its way into our times mainly through the work of Theodore de Laguna [14] and Alfred N. Whitehead [69]. At first, neither de Laguna nor Whitehead considered *parthood* as a basic concept, trying instead to reduce it to other primitive concepts such as *region* and *connection*.¹⁴

¹⁴ Whitehead actually used the converse of parthood in his earlier attempts to develop a point-free theory of time, or rather events, in [67, 68], which may in fact be

(For instance, Whitehead took ‘ $x \sqsubseteq y$ ’ to mean that every region connected to x is connected to y .) Already in 1929, however, Tarski [54] developed his geometry of solids based on the part-whole relation (along with the primitive *ball*), and many other theories have followed since. Given suitable axiomatizations of the relevant primitives, points are then recovered as higher-order entities in one of the following ways:

- as equivalence classes of convergent series of nested regions (following [69]);
- as concentric sets of balls [54];
- as special classes of filters [24];
- as equivalence classes of limited ultrafilters [48];
- as maximal coincidence sets (or maximal clans) [48, 3].

Properties and relations between regions may then be used to introduce also the topological notion of an *open* or *closed* set or the metrical notion of *equidistance*, and these in turn allow for the development of topological and geometrical theories that do justice to the irreducible extended character of space. As of today, topological Hausdorff spaces ([24]), locally compact Hausdorff spaces ([48]), and Euclidean geometry ([54, 21]), to mention just few, have been reconstructed in this way. (See [19, 62] for further details and overviews.)

This line of research is addressed by three contributions to the present issue. In *Part I*, Geoffrey Hellman and Stewart Shapiro’s “Regions-based two-dimensional continua: the Euclidean case” extends their work from [25] to the classical two-dimensional continuum in a system whose primitive notions are *region*, *parthood*, and *congruence*, along with additional *direction* primitives. Cristina Coppola and Giangiacomo Gerla’s “Mereological foundations of point-free geometry via multi-valued logic” draws instead a non-standard picture of the point-free landscape in which the underlying logic is multi-valued and vagueness is regarded to be an inherent feature of the objects that ground our spatial intuitions. In addition, Klaus Robering’s contribution to *Part II*, “The whole is greater than the part”, analyzes the tacit mereogeometrical and mereotopological assumptions that play an essential role in the proofs offered by Euclid in his *Elements*.

regarded as one of the first mereological theories developed independently of Husserl’s and Leśniewski’s projects.

4.3. Mereology in metaphysics and epistemology

A third area of research, which has attracted enormous attention in the past few years, belongs more squarely to the tradition initiated by Husserl's conception of mereology and deals with questions concerning the metaphysical and epistemological underpinnings of the part-whole relation. We have already mentioned, in this connection, Aaron Cotnoir's and Peter Forrest's contributions to the present issue, which deal with the idempotence assumption underlying the classical conception of parthood ("Abelian mereology", in *Part I*), and specifically with its apparent failure in the domain of universals ("The mereology of structural universals", in *Part II*), respectively. Even more basic, perhaps, is the question addressed by Peter Simons in his "Mereology and truth-making" (*Part II*), which deals with the very idea that not all mereological propositions express general laws: claims of the form " a is part of b ", for instance, may be true as a matter of contingent fact; and if they are only true contingently, what is it — one may ask — that *makes* them true?

The debate on the nature and existence of mereological sums and on the principle of unrestricted composition may be viewed in this light, too. As we have seen, this principle is at the heart of classical mereology; yet most philosophers regard it as unacceptable, for while it may prove convenient for the purpose of providing a mereological counterpart of the set-theoretic notion of "class", it also appears to entail the existence of a large variety of *prima facie* implausible entities composed of parts that have nothing to do with one another. Accordingly, two questions have been driving the debate on such matters:

- What does it take for a plurality of objects to have a mereological sum?
- How does a mereological sum relate, ontologically, to the plurality of things that compose it?

The first is essentially the "special composition question" formulated by Peter van Inwagen [63], and while the challenge it poses is genuinely philosophical, it has a technical side as well. For any principled answer is tantamount to forgoing the axiom of unrestricted composition in favor of some weaker existence postulate, and this gives rise to a whole spectrum of systems strictly weaker than classical mereology (see [23, 41]). The second question has been addressed mainly under the rubric of whether mereology is, in Lewis's phrase, "ontologically innocent" [34, p. 81]: is

a mereological sum something *over and above* the plurality of things that compose it? or is it, in some sense or other, identical to its parts? Here, too, the question reflects a genuine philosophical concern, but its ramifications give rise to a number of technical issues as well, the most obvious of which is that the (one-many) relation of identity corresponding to the second option is bound to be non-standard in some way or other [12]. Moreover, it has been claimed that the option in question is incompatible both with the existence of emergent properties and with a plural analogue of Cantor's Theorem, to the effect that any plurality containing two or more members has more sub-pluralities than members. The papers by Claudio Calosi and Einar Bøhn included in *Part II* address precisely these claims. In "Composition, identity and emergence", Calosi displays an argument *against* the first sort of incompatibility; in "Composition as identity and plural Cantor's theorem", Bøhn offers a detailed argument *in favor* of the second, along with his reasons for thinking that the incompatibility with Cantor's Theorem is, in fact, not a defect but a virtue of the "composition as identity" view.

Last, but not least, a great deal of research has been focusing on the possibility of relaxing one of the most fundamental assumptions implicit in classical mereology (and weakenings thereof), namely, that parthood is a perfectly determinate relation: given any two entities x and y , there is always an objective, determinate fact of the matter as to whether or not x is part of y . There are metaphysical as well as epistemological reasons for questioning such an assumption. And while it may be argued that any *prima facie* counterexamples are merely indicative of a certain *de dicto* indeterminacy accompanying ordinary uses of the parthood *predicate* (see [66, §5]), for a growing number of authors it is the parthood *relation* itself that may suffer from genuine *de re* indeterminacy. This has led to the development of several non-classical mereologies in which parthood undergoes a fuzzification that in many ways parallels the fuzzification of membership in Zadeh's set theory [70]. The "rough mereology" developed by Lech Polkowski and his collaborators [43] is perhaps the most fully articulated theory of this kind. And Polkowski's contribution to this issue, "Mereology and uncertainty", offers a detailed illustration of how rough mereology may be brought to bear on the problem of uncertainty of knowledge, including applications to the calculus of perceptions, mereogeometry, and approximate spatial reasoning.

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