Title : will be set by the publisher
Editors : will be set by the publisher
EAS Publications Series, Vol. ?, 2008

PIERNIK MHD CODE — A MULTI–FLUID, NON–IDEAL EXTENSION OF THE RELAXING–TVD SCHEME (II)

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Abstract. We present a new multi–fluid, grid MHD code PIERNIK, which is based on the Relaxing TVD scheme (Jin and Xin, 1995). The original scheme (see Trac & Pen (2003) and Pen *et al.* (2003)) has been extended by an addition of dynamically independent, but interacting fluids: dust and a diffusive cosmic ray gas, described within the fluid approximation, with an option to add other fluids in an easy way. The code has been equipped with shearing–box boundary conditions, and a selfgravity module, Ohmic resistivity module, as well as other facilities which are useful in astrophysical fluid–dynamical simulations. The code is parallelized by means of the MPI library. In this paper we introduce the multifluid extension of Relaxing TVD scheme and present a test case of dust migration in a two–fluid disk composed of gas and dust. We demonstrate that due to the difference in azimuthal velocities of gas and dust and the drag force acting on both components dust drifts towards maxima of gas pressure distribution.

1 Multifluid extension of the Relaxing TVD scheme

The basic set of conservative MHD equations (see Paper I, (Hanasz et al., 2008), this volume) describes a single fluid. The Relaxing TVD scheme by Pen, Arras & Wong (2003) can be easily extended for multiple fluids by concatenation of the vectors of conservative variables for different fluids

$$\mathbf{u} = \big(\underbrace{\rho^i, m_x^i, m_y^i, m_z^i, e^i}_{\text{ionized gas}}, \underbrace{\rho^n, m_x^n, m_y^n, m_z^n, e^n}_{\text{neutral gas}}, \underbrace{\rho^d, m_x^d, m_y^d, m_z^d}_{\text{dust}}\big), \tag{1.1}$$

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DOI: (will be inserted later)

representing ionized gas, neutral gas, as well as dust treated as a pressureless fluid. In a short notation this can be written as

$$\mathbf{u} = (\mathbf{u}^i, \mathbf{u}^n, \mathbf{u}^d). \tag{1.2}$$

The corresponding fluxes for these fluids are combined in a similar way

$$\mathbf{F}(\mathbf{u}, \mathbf{B}) = \left(\mathbf{F}^i(\mathbf{u}^i, \mathbf{B}), \mathbf{F}^n(\mathbf{u}^n), \mathbf{F}^d(\mathbf{u}^d)\right), \tag{1.3}$$

where the elementary flux vectors like $\mathbf{F}^i(\mathbf{u}^i, \mathbf{B})$, $\mathbf{F}^n(\mathbf{u}^n)$ and $\mathbf{F}^d(\mathbf{u}^d)$ are considered as fluxes computed independently for each fluid in the single fluid description. In multidimensional computations the fluxes $\mathbf{G}(\mathbf{u}, \mathbf{B})$ and $\mathbf{H}(\mathbf{u}, \mathbf{B})$, corresponding to the transport of conservative quantities in y and z–directions, are constructed in a similar manner. The full multifluid system of MHD equations, including source terms

$$\partial_t \mathbf{u} + \partial_x \mathbf{F} = \mathbf{S}(\mathbf{u}), \tag{1.4}$$

is subsequently solved by means of the Relaxing TVD scheme described in Paper I, together with the induction equation coupling magnetic filed $\mathbf{B}$ to the ionized gas component. The term $\mathbf{S}(\mathbf{u})$ includes any source terms (like gravity) corresponding to individual fluids as well as the terms coupling the dynamics of different fluids.

As an example of mutual fluid interaction we consider the environment of protoplanetary discs, where aerodynamic interaction of gas and dust particles takes place. The interaction of the neutral gas and dust components induces the effects of the drag force in the momentum and energy equations for these fluids

$$\vec{S}^d_m = \alpha^{dn} \rho^d \rho^n \left(\vec{v}^n - \vec{v}^d\right), \qquad \vec{S}^n_m = -\vec{S}^d_m, \tag{1.5}$$

$$S^n_e = \alpha^{dn} \rho^d \rho^n \vec{v}_n \cdot \left(\vec{v}^d_x - \vec{v}^n_x\right), \tag{1.6}$$

where

$$\alpha^{dn} = \frac{1}{\rho^n t_{\rm stop}} \tag{1.7}$$

is the inverse product of gas density and particle stopping time. In general α^{dn} depends on properties of dust grains (theirs sizes and bulk densities) and gas, and their relative velocities. For simplicity in the calculations presented we assumed its constant value of 10. Friction forces can be incorporated, when needed, between any pair of fluids.

2 Dust migration in protoplanetary disks

Radial migration of dust particles through gaseous protoplanetary disks is essential for planetesimals formation (see e.g. Johansen *et al.* (2007) and references therein). In this section we present a test example of dust migration in a 2D gaseous disk, under the action of a drag force, coupling the dust and gas components, defined in formula (1.5). We assume that the two–component disk rotates in the gravitational field of a point mass. Initially gas forms a bell–like distribution, with a density

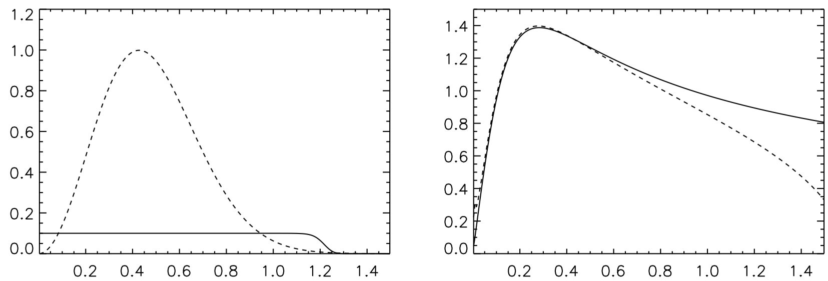

Fig. 1. Left panel: Initial density radial distribution of dust (solid line) and gas (dashed line). Right panel: Initial azimuthal velocity of dust (solid line) and gas (dashed line). Dust velocity is Keplerian, outside the softening radius of point mass gravity. Gas velocity is super–keplerian inside radius 0.4 and sub–keplerian outside due to the gas pressure gradient.

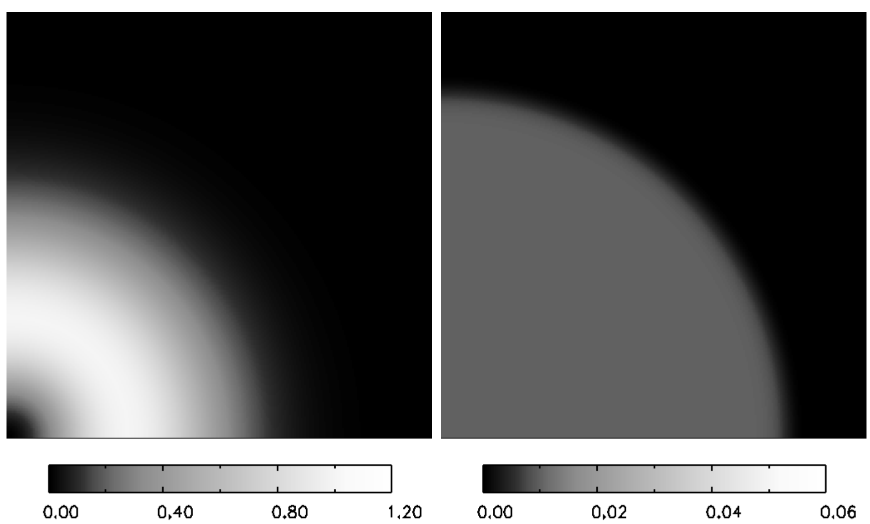

Fig. 2. Initial gas density (left panel) and dust density (right panel) at $t = 0$. A quarter of full disk disk is simulated with the aid of the "corner–periodic" boundary conditions.

maximum in the mid of the disk radius, while the dust component is uniformly distributed across the disk (see Figs. 1 and 2). The simulation is performed in a Cartesian domain with a spatial resolution $n_x \times n_y = 256 \times 256$. To speed up the simulation we use 'corner–periodic' boundary conditions, which enforce 90^o periodicity of simulated objects in the azimuthal angle.

In order to reduce numerical artifacts near the outer domain corner we force gas, outside the disk radius, to follow the assumed rotation curve through additional drag terms added in each timestep to x– and y–components of momentum

$$\Delta\left(\rho v_i\right) = -\alpha_{\text{damp}}\left(v_i - v_{i,\text{init}}\right)\rho\Delta t, \quad i \in \{x, y\} \tag{2.1}$$

where v_i and $v_{i,\text{init}}$ are the current and the initial velocity components respectively, $\alpha_{\text{damp}} \in [0, 1]$ is a factor responsible for artificial damping velocity fluctuations (deviations from the initial velocity distribution) on disk's outskirts.

A radial gradient of gas density in a protoplanetary disks leads to a sub–keplerian rotation. On the other hand dust treated as pressureless fluid tends to orbit the central mass with the Keplerian velocity. As a result, there is a difference between the azimuthal velocities of gas and dust in regions where the radial gradient of gas pressure is non–vanishing. Since dust interacts with gas by means of the friction force, one can expect an exchange of momenta between gas and dust, and thus dust migration in the radial direction (Rice et al., 2004). In the present disc configuration the gas rotation is faster than Keplerian inside the radius of gas density maximum and slower outside, due to the pressure gradient contribution to the radial force balance. In the inner region gas speeds up the dust rotation and in the outer region the dust rotation is slowed down, thus the resulting torques shift dust towards the gas pressure maximum.

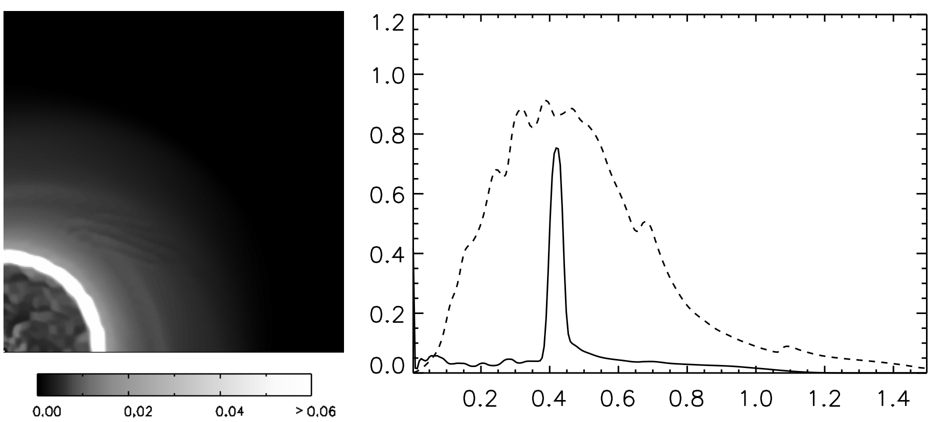

Fig. 3. Left panel: Dust density distribution after 1.5 rotation periods at the radius of the initial maximum gas pressure. The dust component apparently gathers at the radius corresponding to the gas pressure maximum (where density gradient is zero). Right panel: Line plots of radial distribution of dust (solid line) and gas (dashed line). Values for dust are magnified 5 times for plotting on the right panel.

The results of our experiment, performed with the PIERNIK code, are shown in Fig. 3. The effect of dust migration towards the gas pressure maximum is apparent, as expected, since after two rotational periods the majority of dust accumulates in a ring of large density. The simulation presented in this paper demonstrates that the Relaxing TVD scheme can be successfully extended to describe the dynamics of multiple fluids interacting through the drag force.

Acknowledgements

This work was partially supported by Nicolaus Copernicus University through Rector's grants No. 409–A and 516–A, by European Science Foundation within the ASTROSIM project and by Polish Ministry of Science and Higher Education through the grants 92/N–ASTROSIM/2008/0 and PB 0656/P03D/2004/26.

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