Witnessing quantum discord in $2 \times N$ systems

Bogna Bylicka and Dariusz Chruściński
Institute of Physics, Nicolaus Copernicus University,
Grudziądzka 5/7, 87–100 Toruń, Poland

Bipartite states with vanishing quantum discord are necessarily separable and hence positive partial transpose (PPT). We show $2 \times N$ states satisfy additional property: the positivity of their partial transposition is recognized with respect to the canonical factorization of the original density operator. We call such states SPPT (for strong PPT). Therefore, we provide a natural witness for a quantum discord: if a $2 \times N$ state is not SPPT it must contain nonclassical correlations measured by quantum discord. It is an analog of the celebrated Peres-Horodecki criterion: if a state is not PPT it must be entangled.

PACS numbers: 03.65.Ud, 03.67.-a

I. INTRODUCTION

Quantum entanglement is one of the most remarkable features of quantum mechanics and it leads to powerful applications like quantum cryptography, dense coding and quantum computing [1, 2]. However, a quantum state of a composed system may contain other types of nonclassical correlation even if it is separable (not entangled). For a recent 'catalogue' of nonclassical correlations see [3]. The most popular measure of such correlations – quantum discord – was introduced by Ollivier and Zurek and independently by Henderson and Vedral [4, 5]. Hence, quantum discord captures the nonclassical correlations, more general than entanglement, that can exist between parts of a quantum system even if the corresponding quantum entanglement does vanish.

Quantum discord has received much attention in studies involving thermodynamics and correlations [6, 7], complete positivity of reduced quantum dynamics [8, 9] and broadcasting of quantum states [10, 11]. It was shown that quantum discord might be responsible for the quantum computational efficiency of some quantum computation tasks [12–14]. Recently, both Markovian and non-Markovian dynamics of discord was analyzed [15, 16]. Interestingly, contrary to quantum entanglement, Markovian evolution can never lead to a sudden death of discord. Hence, a generic quantum state may keep quantum discord forever. Quantum discord was analytically computed for a class of 2-qubit state [17, 18]. Finally, it was recently generalized for systems with continuous variables to study the correlations in Gaussian states [19, 20]. Interestingly, it was shown [21] that a set of states with vanishing discord has vanishing volume in the set of all states. Actually, this result holds true for any Hilbert space dimension. It shows that a generic state of composed quantum system does contain nonclassical correlation.

In the present paper we analyze a class of states of $2 \times N$ systems. Such 'qubit-quNit' systems play important role in quantum information and were intensively analyzed [26]. Note, that a state with vanishing quantum discord – so called classical-quantum state – is necessarily separable and hence PPT (positive partial transpose). Recently, we introduced [27] a subclass of PPT states – so called SPPT (strong positive partial transpose). These are states where the PPT property is guarantied by the canonical construction based on certain decomposition of the density operator (see below). It was conjectured that all SPPT states are separable. Now, we prove the following result: all classical-quantum $2 \times N$ states are necessarily SPPT. Hence, we provide a natural witness for a quantum discord: if a $2 \times N$ state is not SPPT it must contain nonclassical correlations measured by quantum discord. It is an analog of the celebrated Peres-Horodecki criterion: if a state is not PPT, then it must be entangled.

II. QUANTUM DISCORD

Let us briefly recall the definition of quantum discord [4, 5]. Consider a density operator in $\mathcal{H}_A \otimes \mathcal{H}_B$ and let

$$\mathcal{I}(\rho) = S(\rho_A) + S(\rho_B) - S(\rho) \ , \tag{1}$$

denote the quantum mutual information of a state ρ, where ρ_A (ρ_B) is a reduced density matrix in $\mathcal{H}_A$ ($\mathcal{H}_B$) and $S(\sigma) = -\mathrm{tr}(\rho \log \rho)$ stands for the von Neumann entropy of the density operator σ. Note, that mutual information may be rewritten as follows

$$\mathcal{I}(\rho) = S(\rho_B) - S(\rho|\rho_A) \ , \tag{2}$$

where $S(\rho|\rho_A) = S(\rho) - S(\rho_A)$ denotes quantum conditional entropy. An alternative way to compute the conditional entropy goes as follows: one introduces a measurement on A part defined by the collection of one-dimensional projectors $\{\Pi_k\}$ in $\mathcal{H}_A$ satisfying $\Pi_1 + \Pi_2 + \ldots = \mathbb{I}_A$. The label '$k$' distinguishes different outcomes of this measurement. The state after the measurement when the outcome corresponding to Π_k has been detected is given by

$$\rho_{B|k} = \frac{1}{p_k}(\Pi_k \otimes \mathbb{I}_B)\rho(\Pi_k \otimes \mathbb{I}_B) \ , \tag{3}$$

where $p_k = \mathrm{tr}[\rho_{B|k}(\Pi_k \otimes \mathbb{I}_B)]$. Hence, $\rho_{B|k}$ defines an outcome of the local measurement conditioned on the measurement outcome labeled by 'k'. The entropies $S(\rho_{B|k})$ weighted by probabilities p_k yield to the conditional entropy of part B given the complete measurement $\{\Pi_k\}$ on the part A

$$S(\rho|\{\Pi_k\}) = \sum_k p_k S(\rho_{B|k}) \ . \tag{4}$$

Finally, let

$$\mathcal{I}(\rho|\{\Pi_k\}) = S(\rho_B) - S(\rho|\{\Pi_k\}) \ , \tag{5}$$

be the corresponding measurement induced mutual information. The quantity

$$\mathcal{C}_A(\rho) = \sup_{\{\Pi_k\}} \mathcal{I}(\rho|\{\Pi_k\}) \ , \tag{6}$$

is interpreted [4, 5] as a measure of classical correlations. Now, these two quantities – $\mathcal{I}(\rho)$ and $\mathcal{C}_A(\rho)$ – may differ and the difference

$$\mathcal{D}_A(\rho) = \mathcal{I}(\rho) - \mathcal{C}_A(\rho) \tag{7}$$

is called a quantum discord. For the definition of others discord-like quantities see [24, 25]. Evidently, the above definition is not symmetric with respect to parties A and B. However, one can easily swap the role of A and B to get

$$\mathcal{D}_B(\rho) = \mathcal{I}(\rho) - \mathcal{C}_B(\rho) \ , \tag{8}$$

where

$$\mathcal{C}_B(\rho) = \sup_{\{\widetilde{\Pi}_\alpha\}} \mathcal{I}(\rho|\{\widetilde{\Pi}_\alpha\}) \ , \tag{9}$$

and $\widetilde{\Pi}_\alpha$ is a collection of one-dimensional projectors in $\mathcal{H}_B$ satisfying $\widetilde{\Pi}_1 + \widetilde{\Pi}_2 + \ldots = \mathbb{I}_B$. For a general mixed state $\mathcal{D}_A(\rho) \neq \mathcal{D}_B(\rho)$. However, it turns out that $\mathcal{D}_A(\rho)$, $\mathcal{D}_B(\rho) \geq 0$. Moreover, on pure states, quantum discord coincides with the von Neumann entropy of entanglement $S(\rho_A) = S(\rho_B)$. States with zero quantum discord – so called classical-quantum states – represent essentially a classical probability distribution p_k embedded in a quantum system. One shows that $\mathcal{D}_A(\rho) = 0$ if and only if there exists an orthonormal basis $|k\rangle$ in $\mathcal{H}_A$ such that

$$\rho = \sum_k p_k \, |k\rangle\langle k| \otimes \rho_k^{(B)} \ , \tag{10}$$

where $\rho_k^{(B)}$ are density matrices in $\mathcal{H}_B$. Similarly, $\mathcal{D}_B(\rho) = 0$ if and only if there exists an orthonormal basis $|\alpha\rangle$ in $\mathcal{H}_B$ such that

$$\rho = \sum_\alpha q_\alpha \, \rho_\alpha^{(A)} \otimes |\alpha\rangle\langle\alpha| \ , \tag{11}$$

where $\rho_\alpha^{(A)}$ are density matrices in $\mathcal{H}_A$. It is clear that if $\mathcal{D}_A(\rho) = \mathcal{D}_B(\rho) = 0$, then ρ is diagonal in the product basis $|k\rangle \otimes |\alpha\rangle$ and hence

$$\rho = \sum_{k,\alpha} \lambda_{k\alpha} \, |k\rangle\langle k| \otimes |\alpha\rangle\langle\alpha| \ , \tag{12}$$

is fully encoded by the classical joint probability distribution $\lambda_{k\alpha}$.

In this paper we consider only $\mathcal{D}_A$. Note, that $\Pi_k = |k\rangle\langle k|$ defines a measurement which is optimal for (9). Hence, $\mathcal{D}_A(\rho) = 0$ if

$$\rho = \sum_k (\Pi_k \otimes \mathbb{I}_B)\rho(\Pi_k \otimes \mathbb{I}_B) \ . \tag{13}$$

States with a positive quantum discord do contain nonclassical correlations even if they are separable. Hence nonvanishing quantum discord indicates a kind of quantumness encoded in a separable mixed state. Actually, there is a simple necessary criterion for zero quantum discord [21]: if $\mathcal{D}_A(\rho) = 0$, then

$$[\rho, \rho_A \otimes \mathbb{I}_B] = 0 \ . \tag{14}$$

Hence, if ρ does not commute with $\rho_A \otimes \mathbb{I}_B$ its quantum discord is strictly positive and, hence, ρ is nonclassically correlated. This quantumness may we associated for example to the impossibility of local broadcasting [10, 11]. For the recent discussion of zero discord states see [22, 23].

III. MAIN RESULT

Any state of a bipartite system living in $\mathbb{C}^2 \otimes \mathbb{C}^N$ may be considered as a block 2×2 matrix with $N \times N$ blocks. Positivity of ρ implies that $\rho = \mathbf{X}^\dagger \mathbf{X}$ for some 2×2 upper triangular block matrix $\mathbf{X}$ (due to the well known Cholesky decomposition)

$$\mathbf{X} = \left(\begin{array}{c|c} X_1 & SX_1 \\ \hline 0 & X_2 \end{array} \right) \ , \tag{15}$$

with arbitrary $N \times N$ matrices X_1, X_2 and S. One finds

$$\rho = \mathbf{X}^\dagger \mathbf{X} = \left(\begin{array}{c|c} X_1^\dagger X_1 & X_1^\dagger S X_1 \\ \hline X_1^\dagger S^\dagger X_1 & X_1^\dagger S^\dagger S X_1 + X_2^\dagger X_2 \end{array} \right) \ , \tag{16}$$

and for its partial transposition

$$\rho^{\mathrm{T}_A} = \left(\begin{array}{c|c} X_1^\dagger X_1 & X_1^\dagger S^\dagger X_1 \\ \hline X_1^\dagger S X_1 & X_1^\dagger S^\dagger S X_1 + X_2^\dagger X_2 \end{array} \right) \ . \tag{17}$$

Note, that there is a gauge freedom in choosing X_1, X_2 and S: one may perform the following transformation

$$X_1 \longrightarrow G_1 X_1 \ , \quad X_2 \longrightarrow G_2 X_2 \ , \quad S \longrightarrow G_1 S G_1^{-1} \ ,$$

with $G_1, G_2 \in U(N)$, leaving the formula for ρ invariant. In particular, one can always take X_1 to be semipositive definite. Clearly, ρ is PPT iff there exists $\mathbf{Y}$ such that $\rho^{\mathrm{T}_A} = \mathbf{Y}^\dagger \mathbf{Y}$. The choice of $\mathbf{Y}$ (if it exists) is highly nonunique. Note, however, that there is a 'canonical' candidate for $2N \times 2N$ matrix $\mathbf{Y}$ defined by (15) with S replaced by $S^\dagger$, that is

$$\mathbf{Y} = \left(\begin{array}{c|c} X_1 & S^\dagger X_1 \\ \hline 0 & X_2 \end{array} \right) \ , \tag{18}$$

and hence

$$\mathbf{Y}^\dagger \mathbf{Y} = \left(\begin{array}{c|c} X_1^\dagger X_1 & X_1^\dagger S^\dagger X_1 \\ \hline X_1^\dagger S X_1 & X_1^\dagger S S^\dagger X_1 + X_2^\dagger X_2 \end{array} \right) \ . \tag{19}$$

Let us observe that if S is normal, that is,

$$S^\dagger S = S S^\dagger \ , \tag{20}$$

then $\rho^{T_A} = \mathbf{Y}^\dagger \mathbf{Y}$ and hence ρ is PPT. We call such PPT states — SPPT states [27]. Note, that condition (20) is gauge invariant, that is, if S satisfies (20) so does $S' = G_1 S G_1^{-1}$. For a generalization of SPPT for $M \times N$ systems cf. [27].

The main result of our paper consists in the following

Theorem 1 *If $\mathcal{D}_A(\rho) = 0$, then ρ is SPPT.*

To prove it let us observe that $\mathcal{D}_A(\rho) = 0$ implies that there exists a basis $\{f_1, f_2\}$ in $\mathbb{C}^2$ such that

$$\rho = \sum_{i=1}^{2} |f_i\rangle\langle f_i| \otimes \sigma_i \ , \tag{21}$$

where $\sigma_i \geq 0$ and $\mathrm{Tr}(\sigma_1 + \sigma_2) = 1$. Let U be a unitary in $\mathbb{C}^2$ and let $|f_i\rangle = U|e_i\rangle$. The block structure of (21) in the canonical computational basis $\{e_1, e_2\}$ reads as follows

$$\rho = \left(\begin{array}{c|c} \rho_{11} & \rho_{12} \\ \hline \rho_{21} & \rho_{22} \end{array} \right) \ , \tag{22}$$

where

$$\begin{aligned} \rho_{11} &= |U_{11}|^2 \sigma_1 + |U_{12}|^2 \sigma_2 \ , \\ \rho_{22} &= |U_{21}|^2 \sigma_1 + |U_{22}|^2 \sigma_2 \ , \\ \rho_{12} &= U_{11} \overline{U}_{21}\, \sigma_1 + U_{12} \overline{U}_{22}\, \sigma_2 \ , \end{aligned} \tag{23}$$

and $\rho_{21} = \rho_{12}^\dagger$. One has therefore

$$X_1^\dagger X_1 = |U_{11}|^2 \sigma_1 + |U_{12}|^2 \sigma_2 \ , \tag{24}$$

and hence one may take

$$X_1 = \left(|U_{11}|^2 \sigma_1 + |U_{12}|^2 \sigma_2 \right)^{1/2} \ . \tag{25}$$

Clearly, X_1 is hermitian and semipositive definite $X_1 \geq 0$. Assume now that X_1 is full rank $N \times N$ matrix, that is, X_1 is strictly positive. Then

$$X_1^\dagger S X_1 = U_{11} \overline{U}_{21} \sigma_1 + U_{12} \overline{U}_{22} \sigma_2 \ , \tag{26}$$

gives rise to the following formula for S

$$S = X_1^{-1} \left(U_{11} \overline{U}_{21} \sigma_1 + U_{12} \overline{U}_{22} \sigma_2 \right) X_1^{-1} \ . \tag{27}$$

If X_1 is not strictly positive we may take the generalized inverse (so called Moore-Penrose pseudoinverse). Finally, taking into account

$$U_{11} \overline{U}_{21} + U_{12} \overline{U}_{22} = 0 \ , \tag{28}$$

one obtains

$$S = U_{11} \overline{U}_{21}\, X_1^{-1} \left(\sigma_1 - \sigma_2 \right) X_1^{-1} \ . \tag{29}$$

Note, that since $X_1^{-1} \left(\sigma_1 - \sigma_2 \right) X_1^{-1}$ is hermitian, S is normal which ends the proof.

Corollary 1 *If a PPT state ρ in $\mathbb{C}^2 \otimes \mathbb{C}^N$ is not SPPT, then the quantum discord of ρ does not vanish.*

Remark 1 It turns out that this result does not hold for general $M \times N$ systems with $M > 2$. Consider for example $M = 3$. One can easily introduce $3 \times N$ SPPT states as follows [27]: let $\rho = \mathbf{X}^\dagger \mathbf{X}$ for some 3×3 upper triangular block matrix $\mathbf{X}$

$$\mathbf{X} = \left(\begin{array}{c|c|c} X_1 & S_{12}X_1 & S_{13}X_1 \\ \hline 0 & X_2 & S_{23}X_2 \\ \hline 0 & 0 & X_3 \end{array} \right) , \tag{30}$$

with arbitrary $N \times N$ matrices X_1, X_2, X_3 and S_{12}, S_{13}, S_{23}. Now, ρ is SPPT [27] if all three matrices S_{kl} are normal and

$$S_{12}S_{13}^\dagger = S_{13}^\dagger S_{12} \ . \tag{31}$$

One easily finds for the block structure

$$\rho = \left(\begin{array}{c|c|c} \rho_{11} & \rho_{12} & \rho_{13} \\ \hline \rho_{21} & \rho_{22} & \rho_{23} \\ \hline \rho_{31} & \rho_{32} & \rho_{33} \end{array} \right) , \tag{32}$$

where

$$\begin{aligned} \rho_{11} &= X_1^\dagger X_1 \ , \\ \rho_{1k} &= X_1^\dagger S_{1k} X_1 \ , \qquad k = 2,3 \ , \\ \rho_{22} &= X_1^\dagger S_{12}^\dagger S_{12} X_1 + X_2^\dagger X_2 \ , \\ \rho_{23} &= X_1^\dagger S_{12}^\dagger S_{13} X_1 + X_2^\dagger S_{23} X_2 \ , \\ \rho_{33} &= X_1^\dagger S_{13}^\dagger S_{13} X_1 + X_2^\dagger S_{23}^\dagger S_{23} X_2 + X_3^\dagger X_3 \ . \end{aligned} \tag{33}$$

Now, $\mathcal{D}_A(\rho) = 0$ if there exists an orthonormal basis $\{f_1, f_2, f_3\}$ in $\mathbb{C}^3$ such that

$$\rho = \sum_{k=1}^{3} |f_k\rangle\langle f_k| \otimes \sigma_k \ , \tag{34}$$

with $\sigma_k \geq 0$ and $\mathrm{Tr}(\sigma_1 + \sigma_2 + \sigma_3) = 1$. Let U be a unitary operator defined by $|e_k\rangle = U|f_k\rangle$. One has

$$\rho_{kl} = \sum_{m=1}^{3} U_{km}\overline{U}_{lm}\sigma_m \ . \tag{35}$$

Therefore formula (33) gives for ρ_{11}

$$X_1^\dagger X_1 = |U_{11}|^2\sigma_1 + |U_{12}|^2\sigma_2 + |U_{13}|^2\sigma_3 \ , \tag{36}$$

and hence one may take

$$X_1 = \left(|U_{11}|^2\sigma_1 + |U_{12}|^2\sigma_2 + |U_{13}|^2\sigma_3 \right)^{1/2} \ . \tag{37}$$

Clearly, X_1 is hermitian and semipositive definite $X_1 \geq 0$. Assume now that X_1 is strictly positive. Then formula (33) gives the following formula for S_{12}

$$S_{12} = X_1^{-1} \left(U_{11}\overline{U}_{21}\sigma_1 + U_{12}\overline{U}_{22}\sigma_2 + U_{13}\overline{U}_{23}\sigma_3 \right) X_1^{-1} \ .$$

Now, contrary to S defined by (27) S_{12} needs not be normal. Using

$$U_{11}\overline{U}_{21} + U_{12}\overline{U}_{22} + U_{13}\overline{U}_{23} = 0 \ , \tag{38}$$

one obtains

$$S_{12} = \lambda_1 H_1 + \lambda_2 H_2 \ , \tag{39}$$

where the complex numbers λ_k are defined by

$$\lambda_1 = U_{12}\overline{U}_{32} \ , \quad \lambda_2 = U_{13}\overline{U}_{33} \ ,$$

and Hermitian operators H_1 and H_2 reads as follows

$$H_1 = X_1^{-1}(\sigma_2 - \sigma_1)X_1^{-1} \ , \quad H_2 = X_1^{-1}(\sigma_3 - \sigma_1)X_1^{-1} \ .$$

Hence

$$[S_{12}, S_{12}^\dagger] = (\lambda_1\overline{\lambda}_2 - \lambda_2\overline{\lambda}_1)\,[H_1, H_2] \ , \tag{40}$$

which shows that in general the commutator $[S_{12}, S_{12}^\dagger]$ does not vanish and hence S_{12} is not normal.

IV. EXAMPLE – X-STATES

To illustrate our analysis let us consider so called X-state of two qubits [17, 18]

$$\rho = \left(\begin{array}{cc|cc} a_{11} & \cdot & \cdot & a_{12} \\ \cdot & b_{11} & b_{12} & \cdot \\ \hline \cdot & b_{21} & b_{22} & \cdot \\ a_{21} & \cdot & \cdot & a_{22} \end{array} \right) , \tag{41}$$

where to make the picture more transparent we replaced all zeros by dots. The matrices

$$a = \left(\begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right) , \quad b = \left(\begin{array}{cc} b_{11} & b_{12} \\ b_{21} & b_{22} \end{array} \right) , \tag{42}$$

satisfy: $a \geq 0$, $b \geq 0$ and $\mathrm{Tr}(a + b) = 1$. Clearly, if $a_{12} = b_{12} = 0$, a state is separable with $\mathcal{D}_A(\rho) = 0$.

If only one off-diagonal element a_{12} or b_{12} is different from zero, then ρ is necessarily entangled being NPT. Hence, let us assume that both $a_{12} \neq 0$ and $b_{12} \neq 0$. Note that partially transposed state ρ^{T_A} has again an X-structure with matrices a and b replaced by

$$\widetilde{a} = \left(\begin{array}{cc} a_{11} & b_{21} \\ b_{12} & a_{22} \end{array} \right) \ , \quad \widetilde{b} = \left(\begin{array}{cc} b_{11} & a_{21} \\ a_{12} & b_{22} \end{array} \right) \ . \tag{43}$$

Hence, X-state ρ is PPT iff $\widetilde{a} \geq 0$ and $\widetilde{b} \geq 0$. Now, positivity of ρ is equivalent to

$$a_{11}a_{22} \geq |a_{12}|^2 \ , \quad b_{11}b_{22} \geq |b_{12}|^2 \ . \tag{44}$$

A state is PPT if additionally one has

$$a_{11}a_{22} \geq |b_{12}|^2 \ , \quad b_{11}b_{22} \geq |a_{12}|^2 \ . \tag{45}$$

One shows [27] that a state is SPPT iff

$$|a_{12}| = |b_{12}| \ . \tag{46}$$

Clearly, (46) implies (45). Finally, following our analysis it is easy to show that ρ has vanishing discord iff it satisfies (46) and

$$a_{11} = b_{22} \ , \quad a_{22} = b_{11} \ . \tag{47}$$

Hence, $\mathcal{D}_A(\rho) = 0$ if and only if matrices $||a_{ij}||$ and $||b_{ij}||$ are unitarily equivalent $b = VaV^\dagger$ with

$$V = \left(\begin{array}{cc} 0 & e^{i\mu} \\ e^{i\nu} & 0 \end{array} \right) \ , \quad \mu, \nu \in \mathbb{R} \ . \tag{48}$$

Therefore, one has the following chain of proper inclusions

$$\{\mathcal{D}_A = 0\} \subset \mathrm{SPPT} \subset \mathrm{PPT} \ .$$

In particular if ρ is Bell diagonal, i.e.

$$\begin{aligned} a_{11} &= a_{22} = p_1 + p_2 \ , \\ a_{12} &= p_1 - p_2 \ , \\ b_{11} &= b_{22} = p_3 + p_4 \ , \\ b_{12} &= p_3 - p_4 \ , \end{aligned}$$

where $p_k \geq 0$ and $p_1 + p_2 + p_3 + p_4 = 1$, then ρ is SPPT if $|p_1 - p_2| = |p_3 - p_4|$. Moreover, $\mathcal{D}_A(\rho) = 0$ if and only if 1) $p_1 = p_3$ and $p_2 = p_4$, or 2) $p_1 = p_4$ and $p_2 = p_3$. Hence, discord zero Bell diagonal state of 2 qubits has the following form

$$\rho = \frac{1}{4} \left(\begin{array}{cc|cc} 1 & \cdot & \cdot & q \\ \cdot & 1 & \pm q & \cdot \\ \hline \cdot & \pm q & 1 & \cdot \\ q & \cdot & \cdot & 1 \end{array} \right) \ , \tag{49}$$

where $-1 \leq q \leq 1$. This results do agree with the analysis of X-state performed in [17] and recently in [18]. Note, that the above formula defines 1-dimensional subset in the 3-dimensional set of Bell-diagonal states. Let us observe that for Bell diagonal states $\rho_A = \rho_B = \mathbb{I}_2/2$ and hence the condition (14) is satisfied for all Bell diagonal states. It shows that a necessary criterion of zero quantum discord [21] cannot detect discord within this class. Note, that (49) implies that any convex combination $\frac{1}{2}(P_1 + P_2)$ of arbitrary two Bell projectors P_1 and P_2 has vanishing discord.

V. CONCLUSIONS

We provided a simple witness for a nonclassical correlations measured by a quantum discord in $2 \times N$ systems. We stress that our result is not true for $M \times N$ system with $M > 2$. Note the similarity with Peres-Horodecki criterion. Being PPT is equivalent to separability only for 2×2 and 2×3 systems. It would be interesting to look for the condition $\mathcal{D}_A(\rho) = 0$ for the generalization of X-states for $d \times d$ system. Such states were constructed in [28] (we called them circulant states, see also [29]). In particular they provide generalization of Bell diagonal states of two qudits.

Acknowledgments. This work was partially supported by the Polish Ministry of Science and Higher Education Grant No 3004/B/H03/2007/33.

[1] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, (Cambridge University Press, Cambridge, England, 2000).

[2] R. Horodecki, P. Horodecki, M. Horodecki and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).

[3] K. Modi, T. Paterek, W. Son, V. Vedral, and M.

Williamson, Phys. Rev. Lett. **104**, 080501 (2010).
[4] H. Ollivier and W. H. Żurek, Phys. Rev. Lett. **88**, 017901 (2001).
[5] L. Henderson and V. Vedral, J. Phys. A: Math. Gen. **34**, 6899 (2001).
[6] W. H. Zurek, Phys. Rev. A **67**, 012320 (2003).
[7] M. Horodecki, P. Horodecki, R. Horodecki, J. Oppenheim, A. Sen(De), U. Sen, and B. Synak-Radtke, Phys. Rev. A **71**, 062307.
[8] C. A. Rodriguez-Rosario, K. Modi, A. Kuah, A. Shaji, and E. C. G. Sudarshan, J. Phys. A **41**, 205301 (2008).
[9] A. Shabani and D. A. Lidar, Phys. Rev. Lett. **102**, 100402 (2009)
[10] M. Piani, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **100**, 090502 (2008).
[11] M. Piani, M. Christandl, C. E. Mora, and P. Horodecki, Phys. Rev. Lett. **102**, 250503 (2009).
[12] A. Datta, A. Shaji, and C. Caves, Phys. Rev. Lett. **100**, 050502 (2008).
[13] B. P. Lanyon, M. Barbieri, M. P. Almeida, and A. G. White, Phys. Rev. Lett. **101**, 200501 (2008).
[14] A. Datta and S. Gharibian, Phys. Rev. A **79**, 042325 (2009).
[15] T. Werlang, S. Souza, F.F. Fanchini, and C.J. Villas-Boas, Phys. Rev. A **80**, 024103 (2009).
[16] F. F. Fanchini, T. Werlang, C.A. Brasil, L.G.E. Arruda, and A.O. Caldeira, arXiv:0911.1096.
[17] S. Luo, Phys. Rev. A **77**, 042303 (2008).
[18] Mazhar Ali, A. R. P. Rau, Gernot Alber, *Quantum discord for two-qubit X-states*, arXiv:1002.3429.
[19] P. Giorda and M.G.A. Paris, *Gaussian quantum discord*, arXiv:1003.3207.
[20] G. Adesso and A. Datta, *Quantum versus classical correlations in Gaussian states*, arXiv:1003.4979.
[21] A. Ferraro, L. Aolita, D. Cavalcanti, F. M. Cucchietti, and A. Acin, *Almost all quantum states have non-classical correlations*, arXiv:0908.3157.
[22] A. Datta, *A Condition for the Nullity of Quantum Discord*, arXiv:1003.5256.
[23] B. Dakic, V. Vedral, and C. Brukner, *Necessary and sufficient condition for non-zero quantum discord*, arXiv:1004.0190.
[24] A. Brodutch and D.R. Terno, *Quantum discord and local demons*, arXiv:1002.4913.
[25] S. Wu, U.V. Poulsen, and K. Mølmer, Phys. Rev. A **80**, 032319 (2009).
[26] B. Kraus, J. I. Cirac, S. Karnas and M. Lewenstein, Phys. Rev. A **61**, 062302 (2000).
[27] D. Chruściński, J. Jurkowski and A. Kossakowski, Phys. Rev. A **77**, 022113 (2008).
[28] D. Chruściński, and A. Kossakowski, Phys. Rev. A **76**, 032308 (2007).
[29] D. Chruściński and A. Pittenger, J. Phys. A: Math. Theor. **41** (2008) 385301.